Relationships and Calculus Unit

Polynomials

A **polynomial** is a sum of powers of x. The **degree** (also known as **order**) of the polynomial refers to the highest power of x in the polynomial. For example:

- $3x^2 - 4x + 2$ is a polynomial of degree 2, also known as a **quadratic**.
- $5x^3 - 4x^2 + 7x - 1$ and $y^3 - 11x + 2$ are polynomials of degree 3, known as **cubics**.
- $x^4 + 3x^2 + 1$ is a polynomial of degree 4, also known as a **quartic**.
- $3a^5 - 2a + 6$ is a polynomial of degree 5.

The **coefficient** of a term in the polynomial is the number in front of the letter. e.g. in $x^4 + 2x^2 - 7x$, the coefficient of x is –7, the coefficient of x^2 is 2 and the coefficient of x^4 is 1.

In this unit, we factorise cubic and quartic expressions, and solve cubic and quartic equations. The general form of a cubic is $ax^3 + bx^2 + cx + d$ and for a quartic it is $ax^4 + bx^3 + cx^2 + dx + e$.

Roots and Factors

The **roots** of any polynomial $f(x)$ are the solutions to the equation $f(x) = 0$, and are also the numbers where the graph of $y = f(x)$ goes through the x-axis.

Fact
If $x = a$ is a root of a polynomial, then $(x - a)$ must be a factor of the expression.

Where a root is a fraction, $x = \frac{a}{b}$, then the factor can be written $(bx - a)$.

Roots are numbers, factors are brackets.

Root	Corresponding Factor
$x = 5$	$(x - 5)$
$x = -4$	$(x + 4)$
$x = 0$	x
$x = \frac{1}{2}$	$(2x - 1)$
$x = -\frac{5}{7}$	$(7x + 5)$

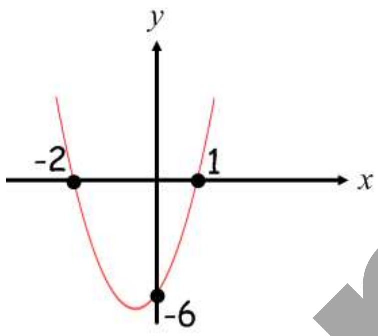
If all of the roots of a polynomial are real, we can work out its equation if we are told (or shown via a graph) all of its roots and at least one other point on the graph.

Fact
If we are told that the roots of a <u>quadratic</u> expression are a and b, then the quadratic must be of the form $y = k(x - a)(x - b)$, for some value of k that needs to be found.

If we are told that the roots of a <u>cubic</u> expression are a, b and c, then the cubic must be of the form $y = k(x - a)(x - b)(x - c)$, for some value of k that needs to be found.

Example 1 – quadratic

The diagram shows the graph of $y = f(x)$, where $f(x)$ is a quadratic function. Express $f(x)$ in the form $f(x) = k(x+a)(x+b)$.



Solution

The diagram shows that one root is $x = -2$, so one of the factors is $(x+2)$. The other root is $x = 1$, so the other factor is $(x-1)$, so the equation can be written $f(x) = k(x+2)(x-1)$.

We are told that the parabola goes through the point (0, –6), so we substitute $x = 0$ and $f(x) = -6$ into the equation:

$$\begin{aligned} f(x) &= k(x+2)(x-1) \\ -6 &= k(0+2)(0-1) && \text{(substituting in } x=0 \text{ and } f(x)=-6) \\ -6 &= k(2)(-1) \\ -6 &= -2k && \text{(simplifying)} \\ k &= \frac{-6}{-2} \\ k &= 3 \end{aligned}$$

Answer: $f(x) = 3(x+2)(x-1)$

The method works in the same way for cubic functions, except there are three factors, so the equation will be of the form $y = k(x-a)(x-b)(x-c)$ for roots a, b and c.

Example 2 – cubic

A cubic function has roots 2, 4 and –1 and its graph goes through the point (3, 8). Determine the equation of the cubic in the form $y = k(x-a)(x-b)(x-c)$.

Solution

The roots we are given tell us that the equation is of the form $y = k(x-2)(x-4)(x+1)$.

Since we are also told the point (3, 8) is on the graph, we substitute $x = 3$ and $y = 8$ into this equation.

$$\begin{aligned} y &= k(x-2)(x-4)(x+1) \\ 8 &= k(3-2)(3-4)(3+1) && \text{(substituting in } x=3 \text{ and } y=8) \\ 8 &= k(1)(-1)(4) \\ 8 &= -4k && \text{(simplifying)} \\ k &= \frac{8}{-4} \\ k &= -2 \end{aligned}$$

Answer: the cubic has equation $y = -2(x-2)(x-4)(x+1)$

A **repeated root** occurs when two or more factors are the same.

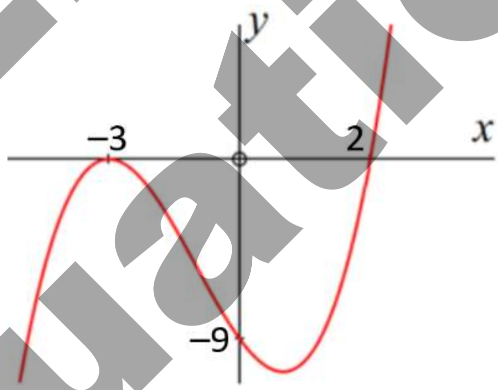
- In the *equation*, this can be seen because the factor has a power, e.g. $(3x+1)^2$ and $(x-5)^3$ are both factors that correspond to repeated roots.
- In a *diagram*, this can be seen because the root will also be a stationary point (see page 105) – e.g. a maximum, minimum or point of inflection. For example the root at $x = -3$ in the diagram for Example 3 is a repeated root.

Facts

- If the graph 'passes through' the *x*-axis from positive to negative (or negative to positive), then the factor has no power.
- If a maximum/minimum stationary point is also a root, then that factor is squared.
- If a point of inflection is also a root, then that factor is cubed.

Example 3 – graph with a repeated root

The diagram shows the graph of $y = g(x)$, where $g(x)$ is a cubic function. Express $g(x)$ in the form $g(x) = k(x+a)(x+b)^2$.



Solution

One root is $x = 2$. The corresponding factor is $(x-2)$.

The root at $x = -3$ is also a maximum stationary point. This means that the corresponding factor of $(x+3)$ is squared.

The formula is $g(x) = k(x+3)^2(x-2)$.

We also know the the graph goes through the point (0, –9), so we substitute $x = 0$ and $g(x) = -9$ into this equation:

$$\begin{aligned} g(x) &= k(x+3)^2(x-2) \\ -9 &= k(0+3)^2(0-2) && \text{(substituting in } x=0 \text{ and } g(x)=-9) \\ -9 &= k(3^2)(-2) \\ -9 &= -18k && \text{(simplifying)} \\ k &= \frac{-9}{-18} = \frac{1}{2} \end{aligned}$$

Answer: $g(x) = \frac{1}{2}(x+3)^2(x-2)$.

Dividing Polynomials

There are two methods to divide polynomials. One is to use the grid method (for multiplication of expressions) backwards. The other is called **synthetic division**.

Synthetic division requires a grid like the one shown on the right. The diagram shows the basic grid for dividing the polynomial $f(x)=ax^3+bx^2+cx+d$ by the factor $(x-n)$. If using the method of synthetic division, the number in the box at the end is the **remainder**.

	x^3	x^2	x	c
n	a	b	c	d
	↓			
				□

Example 1

Divide $(x^3+2x^2-19x+12)\div(x-3)$.

Solution

Method 1 (backwards grid)

Step one: set up a multiplication grid with the divisor $(x-3)$ along the left-hand edge.

Write the highest order term of the dividend (x^3) in the top-left box, and the constant term (12) in the bottom-right box.

x	x^3		
-3			12

Step two: divide to deduce the factors at the top of the left- and right-hand columns:

- $x^3\div x=x^2$
- $12\div(-3)=-4$

	x^2		-4
x	x^3		
-3			12

Step three: multiply to fill in the remaining empty boxes in the left-and right-hand columns:

- $(-3)\times x^2=-3x^2$
- $(-4)\times x=-4x$

	x^2		-4
x	x^3		$-4x$
-3	$-3x^2$		12

Step four: the original dividend contained the terms $2x^2$ and $-19x$, so we need to identify terms that will sum to give these totals.

- $-3x^2+\_\_\_=2x^2$
- $\_\_\_+-4x=19x$

	x^2		-4
x	x^3	$5x^2$	$-4x$
-3	$-3x^2$	$-15x$	12

Step five: deduce the remaining factor in the top row. If it works for both rows, you **must** write "with no remainder" after the table. If it does not work, there is no solution.

	x^2	$-5x$	-4
x	x^3	$5x^2$	$-4x$
-3	$-3x^2$	$-15x$	12

with no remainder

Method 2 (synthetic division)

Step one: set up the table with the coefficients of the dividend in the top row.

We are dividing by $(x-3)$. The corresponding root is 3, so we write 3 on the left-hand side.

	x^3	x^2	x	c
3	1	2	−19	12
	↓			
				□

Step two: 'bring down' the number in the left-hand column.

3	1	2	−19	12
	↓			
	1			□

Step three: multiply the number we have brought down by the root and write it underneath the next coefficient.

3	1	2	−19	12
	↓	3		
	1			□

Step four: add the two numbers in the column together, and then multiply the total by the root.

3	1	2	−19	12
	↓	3	15	
	1	5		□

Repeat step four (bring down and multiply, bring down, multiply bring down…) until the all columns are complete. The number in the final box should be 0 and you **must** write "Remainder = 0".

3	1	2	−19	12
	↓	3	15	−12
	1	5	−4	$\boxed{0}$

Remainder = 0

Final answer: $x^3+2x^2-19x+12\div(x-3)=\underline{\underline{x^2+5x-4}}$.

<u>Example 2</u>

Find the quotient obtained from dividing $\dfrac{2x^3+x^2-13x+6}{2x-1}$.

Solution

Method 1 (backwards grid)

<u>Step one:</u> set up a multiplication grid with the divisor $(2x-1)$ along the left-hand edge.

Write the highest order term of the dividend ($2x^3$) in the top-left box, and the constant term (6) in the bottom-right box.

$2x$	$2x^3$		
-1			6

<u>Step two:</u> divide to deduce the factors at the top of the left- and right-hand columns:

- $2x^3 \div 2x = x^2$
- $6 \div (-1) = -6$

	x^2		-6
$2x$	$2x^3$		
-1			6

<u>Step three:</u> multiply to fill in the remaining empty boxes in the left-and right-hand columns:

- $(-1)\times x^2 = -x^2$
- $(-6)\times 2x = -12x$

	x^2		-6
$2x$	$2x^3$		$-12x$
-1	$-x^2$		6

<u>Step four:</u> the original dividend contained the terms x^2 and $-13x$, so we need to identify terms that will sum to give these totals.

- $-x^2 + \_\_\_ = x^2$
- $\_\_\_ + -12x = -13x$

	x^2		-6
$2x$	$2x^3$	$2x^2$	$-12x$
-1	$-x^2$	$-x$	6

<u>Step five:</u> deduce the remaining factor in the top row. If it works for both rows, you **must** write "with no remainder" after the table. If it does not work, there is no solution.

	x^2	x	-6
$2x$	$2x^3$	$2x^2$	$-12x$
-1	$-x^2$	$-x$	6

with no remainder

Final answer: $\dfrac{2x^3+x^2-13x+6}{2x-1} = \underline{\underline{x^2+x-6}}$.

Method 2 (synthetic division)

<u>Step one:</u> set up the table with the coefficients of the dividend in the top row.

We are dividing by $(2x-1)$. The corresponding root is ½, so we write ½ on the left-hand side.

$$\begin{array}{c|cccc} & x^3 & x^2 & x & c \\ \frac{1}{2} & 2 & 1 & -13 & 6 \\ & \downarrow & & & \\ \hline & & & & \square \end{array}$$

<u>Step two:</u> 'bring down' the number in the left-hand column.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 1 & -13 & 6 \\ & \downarrow & & & \\ \hline & 2 & & & \square \end{array}$$

<u>Step three:</u> multiply the number we have brought down by the root and write it underneath the next coefficient.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 1 & -13 & 6 \\ & \downarrow & 1 & & \\ \hline & 2 & & & \square \end{array}$$

<u>Step four:</u> add the two numbers in the column together, and then multiply the total by the root.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 1 & -13 & 6 \\ & \downarrow & 1 & 1 & \\ \hline & 2 & 2 & & \square \end{array}$$

Repeat step four (bring down and multiply, bring down, multiply bring down…) until the all columns are complete. The number in the final box should be 0 and you **must** write "Remainder = 0".

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 1 & -13 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 2 & 2 & -12 & \boxed{0} \end{array}$$

Remainder = 0

In this method, wen dividing by $(ax+b)$, the final step is to divide by a (2).

Final answer: $\dfrac{2x^2+2x-12}{2} = \underline{\underline{x^2+x-6}}$

Factorising Cubic Polynomials

If a polynomial division by $(x-a)$ has no remainder, then it means that:

- a is a **root** of the equation.
- $(x-a)$ is a **factor** of the polynomial.
- The other factor can be identified from the working in the division table.

This means that twe can factorise a polynomial.

Exam questions will often begin with a part (a) that asks you to 'show that' a bracket is a factor. To show this conclusion, you need to show valid working for a division table, then you need to explicitly say that the remainder is zero.

You would lose a mark is you used the word 'root' when you meant 'factor' or vice versa. Remember that roots are numbers (of the form $x = a$) and factors are expressions in brackets.

Example 1

(a) Show that $(x-1)$ is a factor of x^3+6x^2-x-6.

(b) Hence, factorise x^3+6x^2-x-6 fully.

Solution

(a) We can either use the backwards grid method, or synthetic division.

Method 1 (backwards grid)

Step one: set up a multiplication grid with the divisor $(x-1)$ along the left-hand edge.
Write the highest order term of the dividend (x^3) in the top-left box, and the constant term (–6) in the bottom-right box.

x	x^3		
-1			-6

Steps two-five: complete the table using the method outlined on page 71.

State clearly that there is no remainder.

	x^2	$7x$	6
x	x^3	$7x^2$	$6x$
-1	$-x^2$	$-7x$	-6

with no remainder
$\therefore$ $(x-1)$ is a factor

Method 2 (synthetic division)

Step one: set up the table with the coefficients of the dividend in the top row.
We are dividing by $(x-1)$. The corresponding root is 1, so we write 1 on the left-hand side.

$$\begin{array}{c|cccc}
 & x^3 & x^2 & x & c \\
1 & 1 & 6 & -1 & -6 \\
 & \downarrow & & & \\
\hline
 & & & & \boxed{}
\end{array}$$

Steps two-five: complete the table using the method outlined on page 71.

State clearly that there is no remainder.

$$\begin{array}{c|cccc}
1 & 1 & 6 & -1 & -6 \\
 & \downarrow & 1 & 7 & -6 \\
\hline
 & 1 & 7 & -6 & \boxed{0}
\end{array}$$

Remainder = 0
$\therefore$ $(x-1)$ is a factor

(b) Using the working from the table (circled), we see that the other factor is x^2+7x+6. Therefore:

$$\begin{aligned} x^3+6x^2-x-6 &= (x-1)(x^2+7x+6) \\ &= \underline{\underline{(x-1)(x+1)(x+6)}} \end{aligned}$$

Often an exam question will tell you one of the factors to help you get started. If you are not told a factor, then you have to experiment to find the first factor.

To factorise ax^3+bx^2+cx+d, keep on trying to divide by $(x-n)$ for different factors of d in order (start with $(x-1)$, then $(x+1)$, then $(x-2)$, then $(x+2)$ and so on) until you find any factor that gives a remainder of 0.

Example 2 – requiring experimentation

Factorise $2x^3-15x^2+16x+12$ fully.

Solution

We experiment until we find a factor that gives a remainder of zero. We need to consider factors of 12, so we try $(x\pm1)$, $(x\pm2)$, $(x\pm3)$, $(x\pm4)$, $(x\pm6)$, $(x\pm12)$ in order.

	Method 1 (backwards grid)	**Method 2** (synthetic division)
First try $(x-1)$:	see grid below	see division below

First try $(x-1)$:

Method 1 (backwards grid):

	$2x^2$	X	-12
x	$2x^3$	$-13x$	$-12x$
-1	$-2x^2$	$28x$	12

Does not work, so $(x-1)$ is **NOT** a factor.

Method 2 (synthetic division):

$$\begin{array}{r|rrrr} 1 & 2 & -15 & 16 & 12 \\ & \downarrow & 2 & -13 & 3 \\ \hline & 2 & -13 & 3 & \boxed{15} \end{array}$$

Remainder is 15, so $(x-1)$ is **NOT** a factor.

Now try $(x+1)$:

Method 1 (backwards grid):

	$2x^2$	X	12
x	$2x^3$	$-17x$	$12x$
1	$2x^2$	$4x$	12

Does not work, so $(x+1)$ is **NOT** a factor.

Method 2 (synthetic division):

$$\begin{array}{r|rrrr} -1 & 2 & -15 & 16 & 12 \\ & \downarrow & -2 & 17 & -33 \\ \hline & 2 & -17 & 33 & \boxed{-21} \end{array}$$

Remainder is –21, so $(x+1)$ is **NOT** a factor.

Now try $(x-2)$:

Method 1 (backwards grid):

	$2x^2$	$-11x$	-6
x	$2x^3$	$-11x^2$	$-6x$
-2	$-4x^2$	$22x$	12

With no remainder, so $(x-2)$ **IS** a factor.

Method 2 (synthetic division):

$$\begin{array}{r|rrrr} 2 & 2 & -15 & 16 & 12 \\ & \downarrow & 4 & -22 & -12 \\ \hline & 2 & -11 & -6 & \boxed{0} \end{array}$$

Remainder is 0, so $(x-2)$ **IS** a factor.

Now that we have found a factor, we can complete the factorising in the normal way:

$$\begin{aligned} 2x^3-15x^2+16x+12 &= (x-2)(2x^2-11x-6) \\ &= (x-2)(2x+1)(x-6) \quad \text{or} \quad (2x+1)(x-2)(x-6) \end{aligned}$$

Example 3 – finding an unknown coefficient

Given that $(x+2)$ is a factor of $2x^3+x^2+kx+2$, calculate k.

Solution

We complete a division table using $(x+2)$ as a factor:

Method 1 (backwards grid)

Set up a blank grid using the information given:

x	$2x^3$		
2			2

Complete the grid in the normal way.

	$2x^2$	$-3x$	1
x	$2x^3$	$-3x^2$	x
2	$4x^2$	$-6x$	2

We are told $(x+2)$ is a factor, so we know the grid 'works'. When we collect like terms inside the grid, we get:

$$2x^3+4x^2+(-3x^2)+(-6x)+x+2$$
$$=2x^3+x^2-5x+2$$

The original expression from the question was $2x^3+x^2+kx+2$. Comparing these, we can see that $\underline{\underline{k=-5}}$.

Method 2 (synthetic division)

Set up a blank table using the information given:

$$\begin{array}{r|rrrr} -2 & 2 & 1 & k & 2 \\ & \downarrow & & & \\ \hline & & & & \boxed{} \end{array}$$

Complete the table in the normal way:

$$\begin{array}{r|rrrr} -2 & 2 & 1 & k & 2 \\ & \downarrow & -4 & 6 & -2k-12 \\ \hline & 2 & -3 & k+6 & \boxed{-2k-10} \end{array}$$

The remainder is $-2k-10$.

However since we are told $(x+2)$ is a factor, we know this remainder must also equal 0.

Therefore we can solve an equation:

$$-2k-10=0$$
$$2k=-10$$
$$k=-5$$

Solving Cubic and Quartic Equations

To solve a cubic equation, it must first be factorised, using the method from the last section.

Example 1 – solving cubic equation

(a) Show that $(x-1)$ is a factor of $x^3+6x^2+3x-10$.

(b) Hence, solve the equation $x^3+6x^2+3x-10=0$.

(Solution is on the next page)

Solution (Example 1)

(a) Use a division table using $(x-1)$ as the factor being divided:

Method 1 (backwards grid)

	x^2	$7x$	10
x	x^3	$7x^2$	$10x$
-1	$-x^2$	$-7x$	-10

with no remainder

$\therefore$ $(x-1)$ is a factor ■

Method 2 (synthetic division)

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 3 & -10 \\ & \downarrow & 1 & 7 & 10 \\ \hline & 1 & 7 & 10 & \boxed{0} \end{array}$$

Remainder = 0

$\therefore$ $(x-1)$ is a factor ■

(b) The table from part (a) shows us that the other factor is $x^2+7x+10$. We use this to complete the factorising and then solve:

$$x^3+6x^2+3x-10=0$$
$$(x-1)(x^2+7x+10)=0$$
$$(x-1)(x+5)(x+2)=0$$
$$\underline{\underline{x=1,\ x=-5,\ x=-2}}$$

To solve a *quartic* equation, you will be given one factor and will probably then need to experiment to find a factor of the resulting cubic expression.

Example 2 – solving a quartic equation

(a) Show that $(x+5)$ is a factor of $x^4+2x^3-16x^2-2x+15$.

(b) Hence, or otherwise, solve the equation $x^4+2x^3-16x^2-2x+15=0$, $x\in\mathbb{R}$.

Solution

(a) Use a division table using $(x+5)$ as the factor being divided:

Method 1 (backwards grid)

	x^3	$-3x^2$	$-x$	3
x	x^4	$-3x^3$	$-x^2$	$3x$
5	$5x^3$	$-15x^2$	$-5x$	15

with no remainder

$\therefore$ $(x+5)$ is a factor ■

Method 2 (synthetic division)

$$\begin{array}{r|rrrrr} -5 & 1 & 2 & -16 & -2 & 15 \\ & \downarrow & -5 & 15 & 5 & -15 \\ \hline & 1 & -3 & -1 & 3 & \boxed{0} \end{array}$$

Remainder = 0

$\therefore$ $(x+5)$ is a factor ■

(b) From (a), we know that $x^4+2x^3-16x^2-2x+15=(x+5)(x^3-3x^2-x+3)$.

We now need to factorise the cubic factor x^3-3x^2-x+3.

(continued on next page)

(Example 2 continued)

We experiment until we find a factor that gives a remainder of zero. We need to consider factors of 3, so we try $(x \pm 1)$ and $(x \pm 3)$. On this occasion, $(x-1)$ works:

Method 1 (backwards grid)	**Method 2** (synthetic division)

	x^2	$-2x$	-3
x	x^3	$-2x^2$	$-3x$
-1	$-x^2$	$2x$	3

With no remainder, so $(x-1)$ is a factor.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -1 & 3 \\ & \downarrow & 1 & -2 & -3 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array}$$

Remainder is 0, so $(x-1)$ is a factor.

We can see from the bottom line of the table that the other factor is x^2-2x-3 meaning that we can now complete the factorising in the normal way and use this to solve the equation:

$$\begin{aligned} x^4+2x^3-16x^2-2x+15&=0 \\ (x+5)(x^3-3x^2-x+3)&=0 \\ (x+5)(x-1)(x^2-2x-3)&=0 \\ (x+5)(x-1)(x+1)(x-3)&=0 \\ \underline{x=-5,\ x=1,\ x=-1,\ x=3} \end{aligned}$$

When factorising a cubic expression, it is possible the quadratic bracket cannot be factorised further. When that happens, you should state that there are no solutions to that factor by proving that the discriminant of the quadratic expression is negative (meaning the factor has no real roots).

<u>Example 3 – solving a quartic equation with irreducible root</u>

(a) Show that $(x-6)$ is a factor of $x^4-2x^3-18x^2-32x-24$.

(b) Hence, or otherwise, solve the equation $x^4-2x^3-18x^2-32x-24=0$, $x \in \mathbb{R}$.

Solution

(a) Use a division table using $(x-6)$ as the factor being divided:

Method 1 (backwards grid)	**Method 2** (synthetic division)

	x^3	$4x^2$	$6x$	4
x	x^4	$4x^3$	$6x^2$	$4x$
-6	$-6x^3$	$-24x^2$	$-36x$	-24

with no remainder

$\therefore (x-6)$ is a factor ■

$$\begin{array}{r|rrrrr} 6 & 1 & -2 & -18 & -32 & -24 \\ & \downarrow & 6 & 24 & 36 & 24 \\ \hline & 1 & 4 & 6 & 4 & \boxed{0} \end{array}$$

Remainder = 0

$\therefore (x-6)$ is a factor ■

(continued on next page)

(Example 3 continued)

(b) From (a), we know that $x^4 - 2x^3 - 18x^2 - 32x - 24 = (x-6)(x^3 + 4x^2 + 6x + 4)$.
We now need to factorise the cubic factor $x^3 + 4x^2 + 6x + 4$.

We experiment until we find a factor that gives a remainder of zero. We need to consider factors of 4, so we try $(x \pm 1)$, $(x \pm 2)$ and $(x \pm 4)$. On this occasion, $(x+2)$ succeeds:

Method 1 (backwards grid)

	x^2	$2x$	2
x	x^3	$2x^2$	$2x$
2	$2x^2$	$4x$	4

With no remainder,
so $(x+2)$ is a factor.

Method 2 (synthetic division)

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 6 & 4 \\ & \downarrow & -2 & -4 & -4 \\ \hline & 1 & 2 & 2 & \boxed{0} \end{array}$$

Remainder is 0,
so $(x+2)$ is a factor.

We can see from the bottom line of the table that the other factor is $x^2 + 2x + 2$.

We check the discriminant of $x^2 + 2x + 2$:

$b^2 - 4ac = 2^2 - 4 \times 1 \times 2 = -4$.

$b^2 - 4ac < 0$, so this factor has no real roots.

We can now solve the equation in the normal way:

$$x^4 - 2x^3 - 18x^2 - 32x - 24 = 0$$
$$(x-6)(x^3 + 4x^2 + 6x + 4) = 0$$
$$(x-6)(x+2)(x^2 + 2x + 2) = 0$$

$x = 6$, $x = -2$. No real roots.