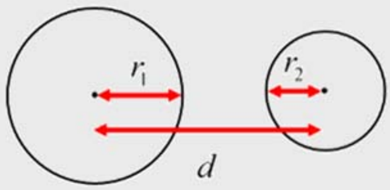
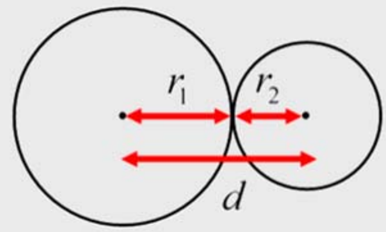
Facts:
If one circle has radius r_1, a second circle has radius r_2 and the distance between their centres is *d*, then:

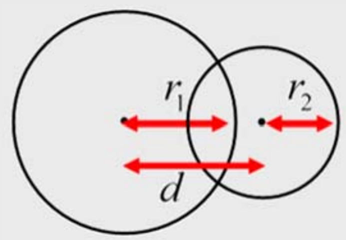
- If $d > r_1 + r_2$ then the two circles **do not intersect.**



- If $d = r_1 + r_2$ then the two circles **intersect in one point.**



- If $d < r_1 + r_2$ then the two circles **have two points of intersection.**



Example 1 – 2016 SQA exam question

Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively. Show that C_1 and C_2 do not intersect.

Solution

C_1 has centre (–5, 6) and radius $r_1 = 3$.

In this equation of C_2: $g = -3$, $f = 0$ and $c = -16$.

Using the usual formulae, its centre is (3, 0) and its radius is $r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{25} = 5$.

The distance between the centres (–5, 6) and (3, 0) is:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-5))^2 + (0-6)^2} \\ &= \sqrt{8^2 + (-6)^2} \\ &= \sqrt{100} = 10 \end{aligned}$$

The sum of the radii is:

$$\begin{aligned} r_1 + r_2 &= 3 + 5 \\ &= 8 \end{aligned}$$

10 > 8, therefore $d > r_1 + r_2$, therefore the circles have <u>no points of intersection</u>.

Example 2

Circles C_1 and C_2 have equations $(x+2)^2+(y-1)^2=8$ and $x^2+y^2-8x-10y+8=0$ respectively. Determine the number of points of intersection of C_1 and C_2.

Solution

C_1 has centre (–2, 1) and radius $r_1=\sqrt{8}$.

In this equation of C_2: $g=-4$, $f=-5$ and $c=8$.

Using the usual formulae, its centre is (4, 5) and its radius is $r_2=\sqrt{g^2+f^2-c}=\sqrt{33}$.

The distance between the centres (–2, 1) and (4, 5) is:

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-(-2))^2+(5-1)^2} \\ &= \sqrt{6^2+4^2} \\ &= \sqrt{52} \approx 7\cdot 2 \end{aligned}$$

The sum of the radii is:

$$\begin{aligned} r_1+r_2 &= \sqrt{8}+\sqrt{33} \\ &\approx 8\cdot 572 \end{aligned}$$

7·2 < 8·572, therefore $d<r_1+r_2$, therefore the circles have two points of intersection.

To find the coordinates of these points of intersection, we would need to be told the equation of a tangent or chord that also went through the point(s) of intersection and then use the method for intersection of a line and a circle (It would not matter which circle equation we used). See Example on page 140 for a reminder of how to find the intersection of a line and a circle.

Note: If one circle is inside the other one, we have to look at the difference between the radii and not the sum. If $d=r_1-r_2$, then the circles touch internally. If $d<r_1-r_2$ then the circles do not touch.

$d=r_1-r_2$ **Touch internally** $d<r_1-r_2$ **do not touch**