Formulae (these <u>are not given</u> on the formula sheet)

The integrating factor for the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is $I(x) = e^{\int P(x)dx}$.

To solve a first order separable differential equation, use the formula $I(x)y = \int I(x)Q(x)dx$.

Example 1 – no initial conditions

Solve $\frac{dy}{dx} + y = 6e^{2x}$. Express your answer in the form $y = f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

On this occasion, it is already in this form, so we do not need to rearrange.

Step two: identify P(x) and Q(x)

$P(x) = 1$ and $Q(x) = 6e^{2x}$.

Step three: find the integrating factor using the formula $I(x) = e^{\int P(x)dx}$.

$$I(x) = e^{\int P(x)dx}$$
$$= e^{\int 1dx} = e^x$$

Step four: use the formula $I(x)y = \int I(x)Q(x)dx$ to find the general solution, and rearrange to make y the subject.

$$I(x)y = \int I(x)Q(x)\ dx$$
$$e^x y = \int e^x \cdot 6e^{2x}\ dx$$
$$e^x y = 6\int e^{3x}\ dx$$
$$e^x y = 6 \cdot \frac{1}{3}e^{3x} + C$$
$$y = 2e^{2x} + \frac{C}{e^x}$$

Sometimes the equation must be rearranged into the standard form before we can solve it.

Example 2 – requires rearranging

Solve $x\frac{dy}{dx} - 2y - x^3 \sin x = 0$. Express your answer in the form $y = f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

$$x\frac{dy}{dx} - 2y - x^3 \sin x = 0$$
$$x\frac{dy}{dx} - 2y = x^3 \sin x$$ (move x^3sinx to the other side)

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