## Motion in a Straight Line

We can apply differentiation and integration to situations involving movement. In these situation the concepts of distance, speed and acceleration are important.

The maths can get quite complicated, so we begin by restricting ourselves to considering the movement of an object moving in 1-dimension along a straight line (usually taken to be the *x*-axis). This is called **rectilinear motion** (or just linear motion).

Because direction matters, we need to use very specific words:

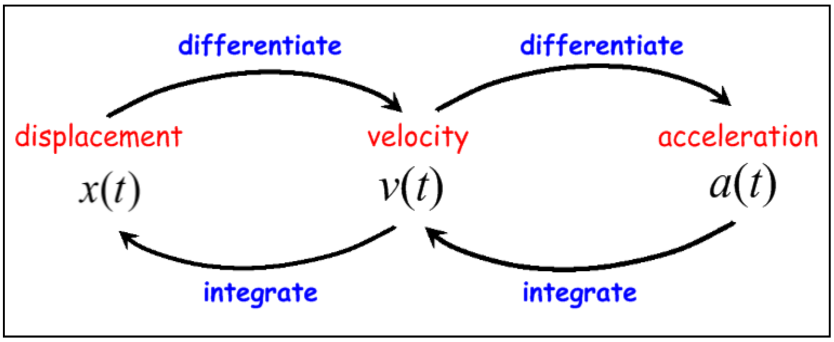
- Instead of talking about the distance travelled, we talk about the **displacement**. Unlike distance, displacement can be positive <u>or</u> negative.
- Instead of talking about the speed, we talk about the **velocity**. Unlike speed, velocity can be positive <u>or</u> negative.
- We also talk about the **acceleration**

| | **Positive** | **Zero** | **Negative** |
|---|---|---|---|
| Displacement | Forward of starting point | At starting point | Behind starting point |
| Velocity | Moving forwards | Not moving | Moving backwards |
| Acceleration | Speeding up | Constant speed | Slowing down |

**Formulae (these <u>are not given</u> on the formula sheet)**

- The **displacement** $x(t)$ is the *x*-coordinate of the particle at time *t*.
- The **velocity** $v(t)$ is the rate of change of displacement with respect to time, i.e. $v = \frac{dx}{dt}$.
- The **acceleration** $a(t)$ is the rate of change of velocity with respect to time, i.e. $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

Velocity may also be written as $\dot{x}(t)$ and acceleration as $\ddot{x}(t)$.




If displacement is measured in metres and time in seconds, then:

- velocity is measured in metres per second, abbreviated m/sec or m/s or $\text{ms}^{-1}$
- and acceleration is measured in metres per second per second, abbreviated $\text{m/s}^2$ or $\text{ms}^{-2}$.

<u>Example 1</u>

**A plane starts from rest. Its velocity in metres per second after *t* seconds is given by $v(t) = \frac{50t}{3t+10}$. Find the acceleration after 12 seconds.**

**Solution**

We differentiate velocity to obtain a formula for acceleration. For this we need to use the quotient rule with $u = 50t$ and $v = 3t + 10$:

$$\begin{aligned} a(t) &= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \\ &= \frac{(3t+10)(50) - (50t)(3)}{(3t+10)^2} \\ &= \frac{150t + 500 - 150t}{(3t+10)^2} \\ &= \frac{500}{(3t+10)^2} \end{aligned}$$

We need to calculate $a(12)$, so we substitute $t = 12$:

$$\begin{aligned} a(12) &= \frac{500}{(3\times 12 + 10)^2} \\ &= \frac{500}{46^2} \quad = \underline{\underline{0\cdot 236 \text{ m s}^{-2}}} \end{aligned}$$

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Example 2

**The displacement of a particle from an origin O, measured in metres, after *t* seconds, is given by the formula $x(t) = 2t^2 - 7t + 3 \quad (t > 0)$.**

**(a) Find the times when the displacement is zero.**

**(b) Find the displacement, velocity and acceleration after 5 seconds.**

**(c) At what time is the particle at rest?**

**Solution**

(a) When the displacement is zero, then $x = 0$, i.e.

$$\begin{aligned} 2t^2 - 7t + 3 &= 0 \\ (2t-1)(t-3) &= 0 \\ t = 0\cdot 5 \ \text{ or } \ t &= 3 \end{aligned}$$

Hence the particle has zero displacement after <u>0·5 seconds and 3 seconds</u>.

(b) Differentiating gives the formula:

$$\begin{aligned} x(t) &= 2t^2 - 7t + 3 \\ v(t) &= x'(t) = 4t - 7 \\ a(t) &= v'(t) = 4 \end{aligned}$$

Substituting $t = 5$ gives into all three formulae:

$$x(5) = 2\times 5^2 - 7\times 5 + 3 = 18 \qquad v(5) = 4\times 5 - 7 = 13 \qquad a(5) = 4$$

Therefore after 5 seconds, the <u>displacement is 18m, the velocity is 13 m s$^{-1}$, and the acceleration is 4 m s$^{-2}$</u>.

(c) If the particle is at rest then $v = 0$. Using the formula for $v(t)$ when $v = 0$ gives:

$$\begin{aligned} 4t - 7 &= 0 \\ t &= \tfrac{7}{4} = 1\cdot 75 \end{aligned}$$

Hence the particle is at rest after <u>1·75 seconds</u>.