Example 1

$$f(x)=\frac{5+x}{x^2(x+3)(x-4)}.$$ **Find the values of x for which $f(x)$ is undefined.**

Solution

Example 2

$g(x)=\sqrt{3x-5}$. **Find the range of values of x for which $g(x)$ is defined.**

Solution

Composite Functions

If you apply one function *f* and then apply another function *g* to the result, you have 'composed' the functions, and the result is called a **composite function**. The order that you apply the functions matters: *f* followed by *g* (written as $g(f(x))$ because the letter closest to *x* is the one that is applied first) is often different to *g* followed by *f* (written $f(g(x))$).

Example 1 – numerical

$f(x)=x^2$ **and** $g(x)=x+1$. **Calculate: (a)** $f(g(3))$ **and (b)** $g(f(3))$.

Solution

Example 2

$f(x)=\sin x$, $g(x)=5x$ **and** $h(x)=x+1$. **Find expressions for:**

(a) $f(g(x))$ **(b)** $g(f(x))$ **(c)** $h(f(x))$ **(d)** $f(h(x))$

Solution

Example 2 – numerical

Calculate the exact value of $\cos 75°$.

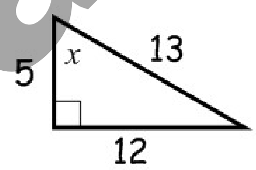
Solution

Trigonometric ratios in a right angled triangle: (not on exam paper)

$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan x = \frac{\text{Opposite}}{\text{Adjacent}}$$

For example in the triangle shown on the right:

$$\sin x = \frac{12}{13}, \quad \cos x = \frac{5}{13} \quad \text{and} \quad \tan x = \frac{12}{5}$$

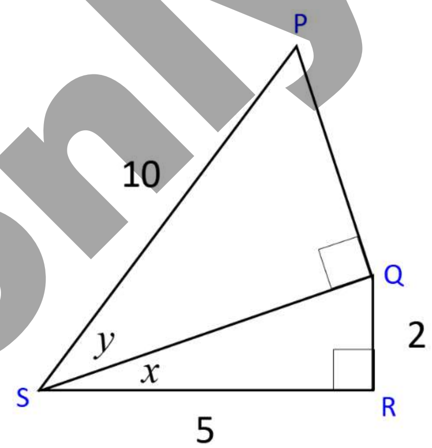


If we had not been told the length of one of the three sides in a right-angled triangle, we could work it out using Pythagoras Theorem.

Example 3 – from a diagram

For the diagram shown, calculate the exact value of $\sin PSR$.

Solution



Maximum and Minimum Values and Sketches of *p*cos*x* + *q*sin*x*

We know from the functions topic (see page 21) that the maximum value of $k\sin(x \pm a)$ or $k\cos(x \pm a)$ is k and the minimum value is $-k$. We can use this information to help us answer other questions.

We can also use the wave function equation to help us sketch a graph of the form *p*cos*x* + *q*sin*x* by considering the rules of graph transformations (see page 32).

Example

(a) Express $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \le a < 360$.

(b) Hence find the maximum value of $5 + \sqrt{3}\sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \le x < 360$.

(c) Hence sketch the graph of $y = \sqrt{3}\sin x° + \cos x° + 1$ for $0 \le x \le 360$.

Solution

Part (a)

Using the standard method from the previous section (working omitted), we can obtain the answer $\sqrt{3}\sin x + \cos x = \underline{2\sin(x + 30)°}$.

Part (b)

Part (c)

Graph Sketching

To sketch the graph of a function, differentiation allows us to identify the shape of the graph because it allows us to identify the coordinates and nature of stationary points.

To sketch the graph of $y = f(x)$, you should include the following features:

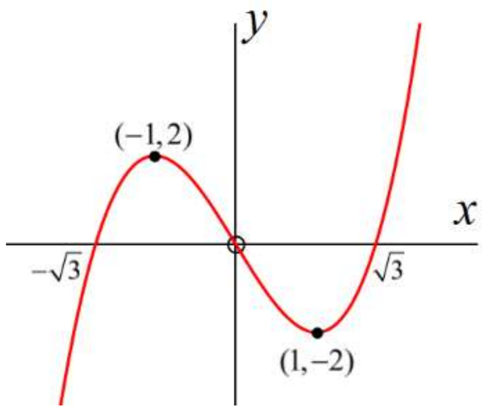
Feature	**How to identify it**
Roots (x-intercepts)	Solve $f(x) = 0$, usually by factorising.
y-intercept	Substitute $x = 0$ into the original equation.
Stationary points (coordinates and nature)	Solve $f'(x) = 0$ and use a nature table.
What happens for very large positive and negative values of x.	Experiment with very large numbers to identify whether the answers get increasingly large and positive, or large and negative.

Example

A function is defined by $f(x) = x^3 - 3x$. Sketch the graph of $y = f(x)$.

Solution

Here is an example of a fully completed sketch:



Example 6

A straight line, L_1, has the equation $2y = 3x + 5$.
A second straight line, L_2, is perpendicular to L_1 and passes through the point (1, 4).
Find the equation of L_2.

Solution

You should also be aware of certain facts involving right-angles in quadrilaterals:

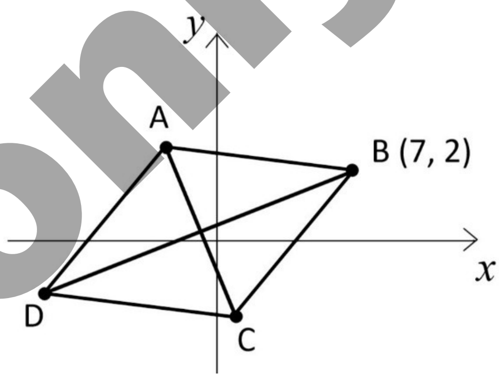
- The corners of **squares** and **rectangles** are right angles (i.e. adjacent sides are perpendicular).
- The diagonals of **squares**, **kites** and **rhombuses** meet at right angles (i.e. the diagonals are perpendicular).

Example 7 – properties of 2 dimensional shapes

The diagram shows a rhombus ABCD.

Diagonal AC has the equation $y = -2x - 5$.

Point B has co-ordinates (7, 2). Find the equation of the diagonal BD.



Solution

Area Between Curves

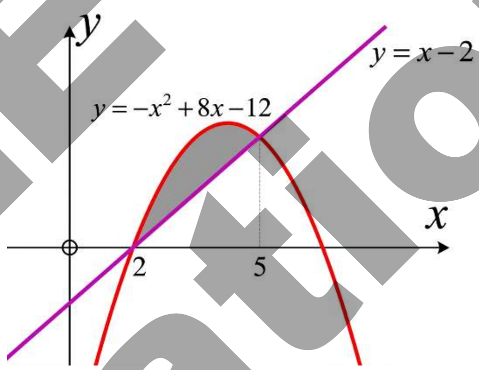
Formula
The area between two curves between the limits $x = a$ and $x = b$ is given by:

$$\int_a^b (\text{upper curve} - \text{lower curve})\, dx$$

Example 1 – limits given

The curve $y = -x^2 + 8x - 12$ and the straight line $y = x - 2$ intersect as shown in the diagram.

Calculate the shaded area between the curves.



Solution