National 5 Mathematics
Revision Notes

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Use this booklet to practise working independently like you will have to in the exam.

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back pages) to look it up.

As you get closer to the final test, you should be aiming to use this booklet less and less.

This booklet is for:

- Students doing the National 5 Mathematics course.
- Students studying one or more of the National 5 Mathematics units: **Expressions and Formulae**, **Relationships** or **Applications.**

This booklet contains:

- The most important facts you need to memorise for National 5 Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:

- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for the exam.

<u>The key to revising for a maths exam is to do questions, not to read notes.</u> **As well as using this booklet, you should also:**

- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or other exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a SmartPhone is on the last page).

Contents

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information becomes available.

We try our hardest to ensure these notes are accurate, but despite our best efforts, mistakes sometimes appear. If you discover any mistakes in these notes, please email us at david@dynamicmaths.co.uk.
An updated copy of the notes will be provided free of charge!
We would like to hear any suggestions you may have for improving our notes.

This version is version 3.0: published December 2018.

Previous versions:
Version 2.1. January 2016.
Version 2.0. July 2015
Version 1.3. October 2014; Version 1.2. January 2014.
Version 1.1. May 2013; Version 1.0. May 2013.

With grateful thanks to **Arthur McLaughlin** and **John Stobo** for their proof reading.

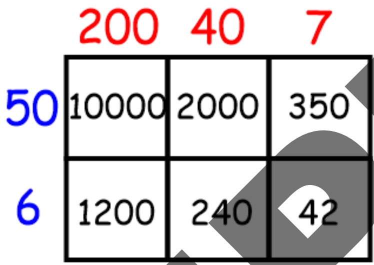
These methods can be extended to any multiplication sum, including multiplication of three- (or more)-digit numbers, or multiplication of decimals.

Example 3 – Multiplying three-digit and two-digit numbers

Multiply 247 × 56

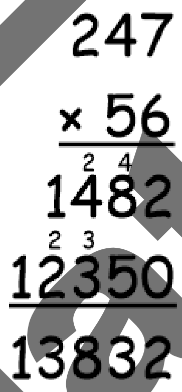
Solution

Using **method A** (grid method) the working is as follows:



10000 + 2000 + 350
+ 1200 + 240 + 42
= 13832

Using **method B** (long multiplication) the working is as follows:



The final answer is £13832

You are also expected to multiply decimal numbers by multiples of 10, 100 or 1000 (e.g. multiplying by 30 or 6000).

The key to this method is splitting into two steps:
- Step A involves multiplying by a single digit using the method above.
- Step B involves multiplying by 10, 100 or 1000 (by moving all the digits to the left).

It does not matter which order Steps A and B are done in. The following table illustrates how this can be done:

Calculation	Step A	Step B
Multiply by 20	Multiply by 2	Multiply by 10
Multiply by 300	Multiply by 3	Multiply by 100

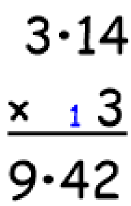
Example 4 – Multiplying by a multiple of 10, 100 or 1000

Multiply without a calculator: 3·14 × 3000.

Solution

First do 3·14 × 3, and then multiply the answer by 1000, (or 3·14 × 1000, and then × 3).

3·14 × 3 = 9·42 *(working with carrying shown on right)*
9·42 × 1000 = 9420



Expressions and Formulae Unit

Surds and Indices

Simplifying Surds

Definition: a **surd** is a square root (or cube root etc.) which does not have an exact answer. e.g. $\sqrt{2} = 1\cdot414213562...$, so $\sqrt{2}$ is a surd. However $\sqrt{9} = 3$ and $\sqrt[3]{64} = 4$, so $\sqrt{9}$ and $\sqrt[3]{64}$ are not surds because they have an exact answer.

We can multiply and divide surds.

Facts

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ $\sqrt{x} \times \sqrt{x} = \left(\sqrt{x}\right)^2 = x$

Example 1

Simplify $3\sqrt{2} \times 5\sqrt{2}$

Solution

$$3\sqrt{2} \times 5\sqrt{2} = 3 \times 5 \times \sqrt{2} \times \sqrt{2}$$
$$= 15 \times 2 \quad \text{(because } \sqrt{2} \times \sqrt{2} = 2\text{)}$$
$$= \underline{\underline{30}}.$$

To simplify a surd, look for square numbers that are factors of the original number.

Examples 2

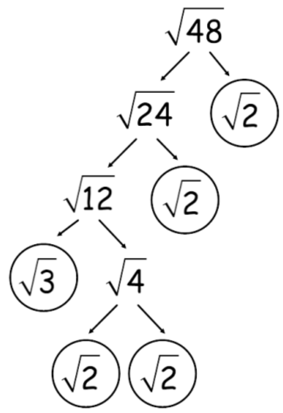
Express $\sqrt{48}$ and $\sqrt{98}$ in their simplest form.

Solution

$$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4 \times \sqrt{3} = \underline{4\sqrt{3}}$$

$$\sqrt{98} = \sqrt{2 \times 49} = \sqrt{2} \times \sqrt{49} = \sqrt{2} \times 7 = \underline{7\sqrt{2}}$$

Alternative Method (for $\sqrt{48}$): split 48 up into its prime factors – see the diagram on the right (note we could split it up however we wanted and would always end up with the same answer).



From the diagram, we get:

$$\sqrt{48} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$$
$$= 2 \times 2 \times \sqrt{3}$$
$$= 4\sqrt{3}$$

Scientific Notation (Standard Form)

Definition: Scientific Notation (also known as Standard Form) is a more efficient way of showing very large or very small numbers.

Definition: a number written normally is said to be "in **normal form**".

- A very large number (one that ends with zeroes e.g. 560 000 or 31 million) will have a *positive* number in the power when written in scientific notation.
- A very small number (one that begins 'zero point…' e.g. 0·02 or 0·00015) will have a *negative* number in the power when written in scientific notation.

Example 1

Express in scientific notation: (a) 92 000 000 (b) 0·000456.

Solution

(a) $9{\cdot}2 \times 10^7$ (b) $4{\cdot}56 \times 10^{-4}$

Example 2

Express in normal form: (a) $3{\cdot}05 \times 10^5$ (b) $3{\cdot}05 \times 10^{-5}$.

Solution

(a) 305 000 (b) 0·0000305

For the exam, you will be expected to use scientific notation in calculations. Make sure you know which buttons to use on your calculator for scientific notation – for instance on some calculators the scientific notation is written EXP and you do not need to type in the '×10'.

Example 2

One light year is approximately $9{\cdot}46 \times 10^{12}$ kilometres.
Calculate the number of metres in 18 light years.
Give your answer in scientific notation.

Solution

First notice that this question asks for the answer to be given in metres, so we must convert from kilometres to metres. We do this by multiplying by 1000.

Our sum is $9{\cdot}46 \times 10^{12} \times 18 \times 1000$

The answer is $1{\cdot}7028 \times 10^{17}$ metres. (units are essential for a fully correct answer – see page 5)

<u>Example 1</u>

Expand and simplify: $(a-7)(a-9)$.

Solution

$(a-7)(a-9)$

$= a\times a \;+\; a\times(-9) \;+\; (-7)\times a \;+\; (-7)\times(-9)$

$= \quad a^2 \qquad -9a \qquad -7a \qquad +63$

$= \underline{a^2-16a+63}$

If you are using the grid method, your completed grid might look like this:

	a	-7
a	$+a^2$	$-7a$
-9	$-9a$	$+63$

<u>Example 2</u>

Expand and simplify: $(2y+3)(y-4)$.

Solution

$(2y+3)(y-4)$

$= 2y\times y \;+\; 2y\times(-4) \;+\; 3\times y \;+\; 3\times(-4)$

$= \quad 2y^2 \qquad -8y \qquad +3y \qquad -12$

$= \underline{2y^2-5y-12}$

If you are using the grid method, your completed grid might look like this:

	$2y$	$+3$
y	$+2y^2$	$+3y$
-4	$-8y$	-12

When squaring a bracket, it is important to realise that (for example) $(x+3)^2$ is <u>NOT</u> x^2+9. Instead you must rewrite $(x+3)^2$ as $(x+3)(x+3)$ and then to multiply out the brackets using the double bracket method.

<u>Example 3</u> – Squaring brackets

Expand and simplify: $(2a+1)^2$.

Solution

$(2a+1)^2 = (2a+1)(2a+1)$

$= 2a\times 2a \;+\; 2a\times 1 \;+\; 1\times 2a \;+\; 1\times 1$

$= \quad 4a^2 \qquad +2a \qquad +2a \qquad +1$

$= \underline{4a^2+4a+1}$

If you are using the grid method, your completed grid might look like this:

	$2a$	$+1$
$2a$	$+4a^2$	$+2a$
$+1$	$+2a$	$+1$

In some questions, the second bracket may have three terms in it (a **trinomial**). In these examples, the basic method (multiply everything in the first bracket by everything in the second bracket) is still the same, however you will have to do more multiplications.

The answer will often involve a term in x^3 (because $x\times x^2 = x^3$).

<u>Example 4</u> – three terms in one bracket

Expand and simplify: $(g+4)(3g^2-2g+5)$.

Example 3

Write $\dfrac{2a^2+a-1}{2a^2+5a-3}$ in its simplest form.

Solution

Factorising both lines gives:

$$\frac{2a^2+a-1}{2a^2+5a-3}=\frac{(2a-1)(a+1)}{(2a-1)(a+3)}$$

$$=\frac{\cancel{(2a-1)}(a+1)}{\cancel{(2a-1)}(a+3)}$$

$$=\frac{(a+1)}{(a+3)}.$$

Do not try to cancel the 'a's. They are not common factors.

Multiplying and Dividing Algebraic Fractions

We multiply and divide fractions in the same way as we do for numerical fractions (shown on page 99). Multiplying fractions is a straightforward procedure – you **multiply the tops** (numerators) **and multiply the bottoms** (denominators).

e.g. $\frac{3}{5}\times\frac{2}{7}=\frac{3\times2}{5\times7}=\frac{6}{35}$ $\qquad$ $\frac{a}{c}\times\frac{b}{c}=\frac{ab}{c^2}$

It is easiest to cancel before you multiply. You can cancel *anything* from the top row with *anything* from the bottom row.

Example 1

Express $\dfrac{a^2}{15b}\times\dfrac{10}{a}$ $\quad a,b\neq0$ as a single fraction in its simplest form.

Solution

Cancelling gives: $\frac{a^{\cancel{2}}}{\cancel{15}_3 b}\times\frac{\cancel{10}^2}{\cancel{a}}=\frac{a}{3b}\times\frac{2}{1}=\frac{2a}{3b}.$ $\qquad$ **Answer**: $\frac{2a}{3b}$

To divide two fractions:

1. Flip the second fraction upside down.
2. Change the sum to be a multiply sum.

e.g.

$$\frac{3}{5}\div\frac{2}{7}=\frac{3}{5}\times\frac{7}{2}=\frac{21}{10}$$

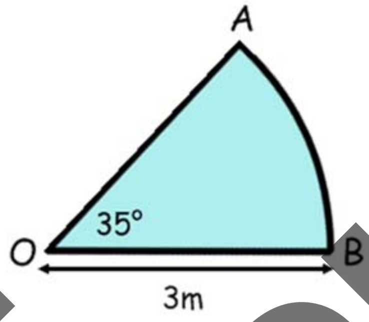
$$\frac{x}{y}\div\frac{a}{x}=\frac{x}{y}\times\frac{x}{a}=\frac{x^2}{ay}$$

Example 2

Express $\dfrac{6b}{ay}\div\dfrac{3ab}{x}$, $\quad a,b,x,y\neq0$ as a single fraction in its simplest form.

Example 2 – Sector area

Calculate the area of sector AOB in this diagram.



Solution

$$A = \frac{35}{360}\pi r^2$$
$$= \pi \times 3^2 \div 360 \times 35$$
$$= 2\cdot 74889357...$$
$$= \underline{2\cdot 75\text{m}^2 \text{ (2 d.p.)}}$$

Note: units for sector area must always be squared units.

Volumes of Solids

You should know from National 4 how to calculate the volume of a **prism**. At National 5 level, you also need to be able to calculate the volume of a **pyramid**. Throughout this topic remember that:

- All volume questions must have answered in cubed units (e.g. m^3, cm^3, $inches^3$).
- You should always state your unrounded answer before rounding (see page 6).

Formula. This formula is **not** given on the National 5 Mathematics exam paper.

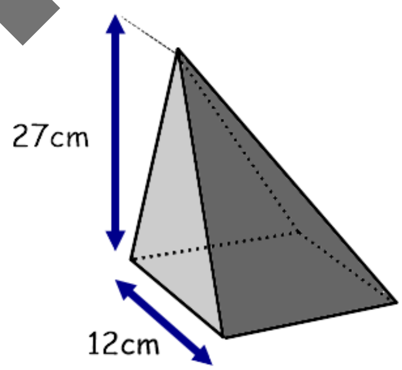
Volume of a Prism:
$$V = Ah$$
$$\text{Volume} = \text{Area of cross section} \times \text{Height}$$

Formula. This formula is given on the National 5 Mathematics exam paper.

Volume of a Pyramid:
$$V = \frac{1}{3}Ah$$
$$\text{Volume} = \frac{1}{3}\text{Area of Base} \times \text{Height}$$

Example 1 – Pyramid

The diagram shows a pyramid with height 27cm and a square base with sides of length 12cm. Calculate the volume of the pyramid.



Solution

The area of a square is given by the formula $A = L^2$, so the area of the base of this pyramid is $12^2 = 144\text{cm}^2$

Therefore, the volume of the whole pyramid is:

$$V = \frac{1}{3}Ah$$
$$= 144 \times 27 \div 3$$
$$= \underline{\underline{1296\text{cm}^3}}$$

Example 2 – inequation

Solve algebraically the inequation $9y+6<4y-34$.

Solution

***Optional step**: write in "invisible plus signs" in front of anything that does not already have a sign*

Step one: move the '+4*y*' over to the left-hand side where it becomes '–4*y*'. Move '+6' to the right-hand side where it becomes '–6'.

Step two: simplify both sides.

Step three: divide to get the final answer.

$$9y+6<4y-34$$
$$+9y+6<+4y-34$$
$$+9y-4y<-34-6$$
$$5y<-40$$
$$y<\frac{-40}{5}$$
$$y<-8$$

Final step: check, by substituting $y = -8$ into both sides of the original equation.

If an equation or an inequation contains a bracket, the bracket can be multiplied out before proceeding with the usual method.

Example 3 – inequation containing brackets

Solve algebraically the inequation $7(x+5)\geq 3x-2$.

Solution

$7(x+5)\geq 3x-2$

$7x+35\geq 3x-2$ (multiplying out brackets)

$+7x+35\geq +3x-2$ (optional: writing in invisible + signs)

$+7x-3x\geq -2-35$ (collecting like terms)

$4x\geq -37$ (simplifying)

$x\geq -\frac{37}{4}$

With inequations, there is one additional rule to learn, which is relevant in step 3, if multiplying or dividing by a negative number.

Rule: In an inequation, if you multiply or divide by a *negative* number, the sign reverses (e.g. $\geq$ becomes $\leq$; $<$ becomes $>$ etc.)

Example 4 – dividing by a negative

Solve algebraically the inequation $x-8>4x+7$.

Solution

$+x-8>+4x+7$

$x-4x>7+8$

$-3x>15$

$x<\frac{15}{-3}$ (changing > to < because we are dividing by a negative number)

$x<-5$

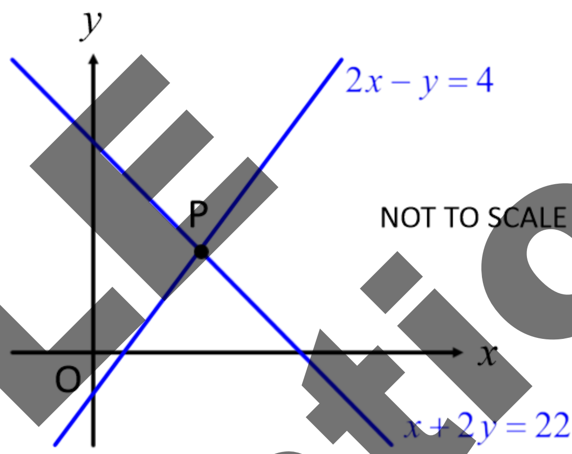
The graphs may already be drawn for you and you may be asked to work out the point at which they intersect. If you cannot see the grid to read the co-ordinate points, then you will need to solve the equation algebraically.

Example 2 – requiring algebra

The graph shows two straight lines with the equations:

- $\mathbf{x + 2y = 22}$
- $\mathbf{2x - y = 4}$

The lines intersect at the point P. Find, algebraically, the coordinates of P.



Solution

To find the point of intersection of two lines, we use simultaneous equations.

This solution uses the subtraction method. However, the adding method could also be used.

$$x + 2y = 22 \quad \times 2$$
$$2x - y = 4 \quad \times 1$$

Multiply the top equation by 2 and the bottom equation by 3:

$$2x + 4y = 44$$
$$2x - y = 4$$

Subtract the equations and then solve:

$$\begin{aligned} \cancel{2x} + 4y &= 44 \\ -\quad \cancel{2x} - y &= 4 \\ \hline 5y &= 40 \\ y &= 8 \end{aligned}$$

Substitute $y = 8$ back into top equation:

$$\begin{aligned} x + 2y &= 22 \\ x + 2 \times 8 &= 22 \\ x &= 22 - 16 \\ x &= 6 \end{aligned}$$

Final Answer: <u>(6, 8).</u>

Changing the Subject of a Formula

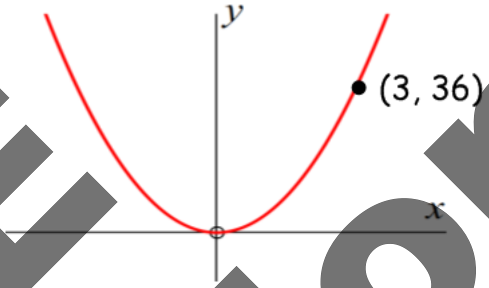
Changing the subject of a formula was introduced at National 4 and the skill is similar to rearranging an equation: you move things from one side to the other and 'do the opposite'.

Changing the subject of a formula is exactly like rearranging an equation: you move things from one side to the other and 'do the opposite'. However, this does not work on every occasion (see example 4)

These values can be substituted back into the equation of the graph. If you don't know the full equation of a graph, they can give you an equation to solve to complete it.

<u>Example</u>

The graph on the right has the equation $y = kx^2$.
The graph passes through the point (3, 36). Find the value of *k*.



Solution

For the point (3 , 36), *x* = 3 and *y* = 36.
Substituting these values into the equation gives:

$$y = kx^2$$
$$36 = k \times 3^2$$
$$36 = k \times 9$$
$$k = 4$$

Roots of Quadratic Equations

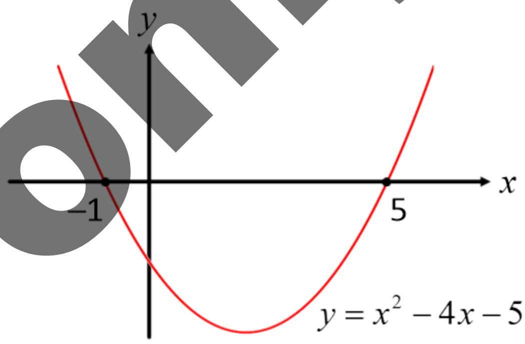
A quadratic equation is of the form $ax^2 + bx + c = 0$. It is essential that the right-hand side of the equation is **zero**. Where this does not occur, it must be rearranged (see Example 5).

> **Definition:** the **roots** of a quadratic equation are another word for its solutions.
> The roots of a graph of an equation are the points that the graph crosses the *x*-axis.

There are three ways of obtaining the roots of a quadratic equation:
1. From a graph *(in practice this doesn't come up often)*.
2. Using factorising *(this is the method you should aim to use wherever possible)*.
3. The quadratic formula *(use this when the equation cannot be factorised)*.

<u>Example 1 – from a graph</u>

The diagram shows the graph of $y = x^2 - 4x - 5$. Using the graph, write down the two solutions of the equation $x^2 - 4x - 5 = 0$.



Solution

The roots are *x* = –1 and *x* = 5.

Factorising is the simplest way of solving a quadratic equation, but you can only use it when the expression can actually be factorised! See page 24 for a reminder of how to factorise.

> **Important** – you <u>must</u> always rearrange the equation so that is has '= 0' on the right-hand side. If you do not do this, you will risk losing **<u>all</u>** the marks.

Example 2 – factorising using double brackets

Solve the equation $2x^2+9x-5=0$.

Solution

Step 1: check that the equation has '= 0' on the right-hand side.
On this occasion, it does, so we do not need to take any further action.

Step 2: factorise the expression.

$$2x^2+9x-5=0$$
$$(2x-1)(x+5)=0$$

Step 3: split up into two separate equations and solve each equation separately.

$$2x-1=0 \qquad x+5=0$$
$$2x=1$$
$$x=\frac{1}{2}, \qquad x=-5$$

Example 3 – factorising with a common factor

Solve the equation $2x^2-6x=0$.

Solution

Step 1: check that the equation has '= 0' on the right-hand side.
On this occasion, it does, so we do not need to take any further action.

Step 2: factorise the expression.

$$2x^2-6x=0$$
$$2x(x-3)=0$$

Step 3: split up into two separate equations and solve each equation separately.

$$2x=0 \qquad x-3=0$$
$$x=0, \qquad x=3$$

Example 4 – Difference of Two Squares

Solve the equation $y^2-49=0$.

Solution

Step 1: check that the equation has '= 0' on the right-hand side.
On this occasion, it does, so we do not need to take any further action.

Step 2: factorise the expression.

$$y^2-49=0$$
$$(y+7)(y-7)=0$$

Step 3: split up into two separate equations and solve each equation separately.

$$y+7=0 \qquad y-7=0$$
$$y=-7, \qquad y=7$$

(Example 3 continued)

Step four: use Pythagoras (or possibly SOH CAH TOA if angles are involved) to calculate another length (or angle) in the triangle.

Using Pythagoras:

$$x^2 = 40^2 - 35^2$$
$$= 375$$
$$x = \sqrt{375}$$
$$= \underline{19 \cdot 4\text{cm}} \text{ (1 d.p.)}$$

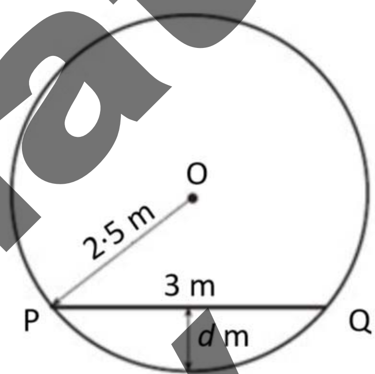
Step five: check whether you have answered the question

You were asked to calculate the width of the table. The width of the table is **OC + the radius**. So in this diagram, the width is 19·4 + 40 = 59·4cm.

Example 4 – Pythagoras in circles (past Intermediate 2 exam question)

The diagram shows the cross-section of a cylindrical oil tank. The centre of the circle is O.
PQ is a chord of the circle. PQ is 3m.
The radius OP is 2·5m.

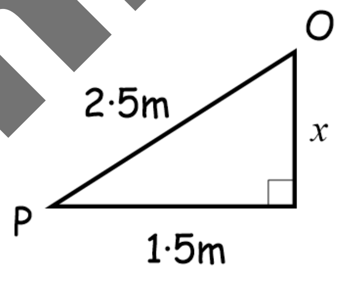
Calculate the depth, *d*, of the oil.



Solution

Steps one, two and three: (see previous example for more detail on how to do these steps) identify the lengths in the right-angled triangle, using the fact that the diagram is symmetrical.

Step four: use Pythagoras (or trigonometry if angles are involved) to calculate another length (or angle) in the triangle.



Using Pythagoras:

$$x^2 = 2 \cdot 5^2 - 1 \cdot 5^2$$
$$= 4$$
$$x = \sqrt{4}$$
$$= \underline{2\text{m}}$$

Step five: check whether you have answered the whole question

In this question, they wanted the **depth of the water**. From the diagram, *d* and *x* add together to make a radius (2·5m). Since *x* = 2 metres, *d* must be 0·5 metres.

Converse of Pythagoras

Pythagoras' Theorem says that if a triangle is right-angled, then $a^2 + b^2 = c^2$.
It can also be used backwards: **if $a^2 + b^2 = c^2$, then the triangle is right-angled**.
This rule is called the **Converse of Pythagoras**, and can be used in a triangle (when all three sides are known) to check whether or not an angle is a right-angle.

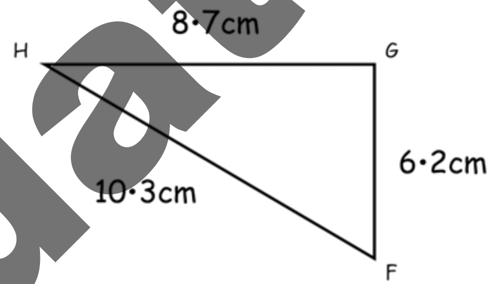
The procedure is:

1. Find the square of the longest side.
2. Find the sum of squares of the two shorter sides.
3. Write a conclusion:
   a. If the two answers are equal, then conclude that the angle is a right angle.
   b. If the two answers are not equal, then conclude that the angle is not a right angle.

Example

Triangle FGH has side lengths as shown in the diagram.

Determine whether triangle FGH is right-angled.



Solution

Step one: calculate the square of the longest side:

$$c^2 = 10 \cdot 3^2 = 106 \cdot 09$$

Step two: calculate the sum of squares of the two shorter sides:

$$a^2 + b^2 = 8 \cdot 7^2 + 6 \cdot 2^2$$
$$= 114 \cdot 13$$

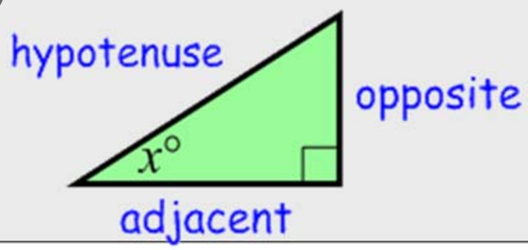
Step three: conclusion:

$8 \cdot 7^2 + 6 \cdot 2^2 \neq 106 \cdot 09^2$, so the triangle is not right-angled.

Revision: Right-Angled Trigonometry (SOH CAH TOA)

At National 4, you learnt to use sine, cosine and tangent in a right-angled triangle. At National 5 you may be expected to remember these skills as part of a longer question. The formulae and basic method are included in this section to remind you of the method.

Trigonometric ratios in a right angled triangle:
Formulae. These formula are **not** given on the National 5 Mathematics exam paper.



$$\sin x^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos x^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan x^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$$

Similarity

Two shapes are similar if they are exactly the same shape but a different size. All their angles will be identical, and all their lengths will be in proportion to each other. One shape will be an enlargement of the other. The factor of the enlargement is called the **scale factor**.

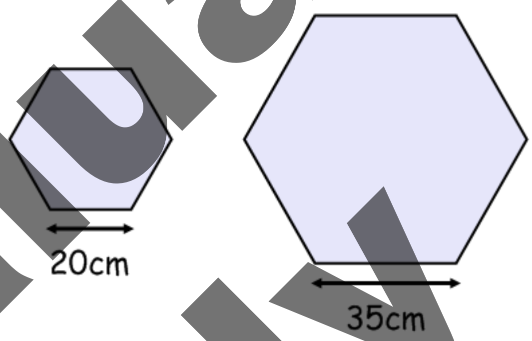
Formula: $\text{Scale Factor} = \dfrac{\text{Length in 'new' shape}}{\text{Length in 'old' shape}}$

At National 5, you need to understand how the scale factor affects area and volume.

Facts
For two similar shapes connected by a scale factor *s*:
- Lengths in the 'new' shape are found by multiplying lengths in the 'old' shape by *s*.
- The area of the 'new' shape is found by multiplying the area of the 'old' shape by s^2.
- The volume of the 'new' shape is found by multiplying the volume of the 'old' shape by s^3.

Example 1 – area

Two hexagons are similar in shape as shown in the diagram. The smaller hexagon has an area of 2500cm². Calculate the area of the larger hexagon.



Solution

Step one: calculate the scale factor.

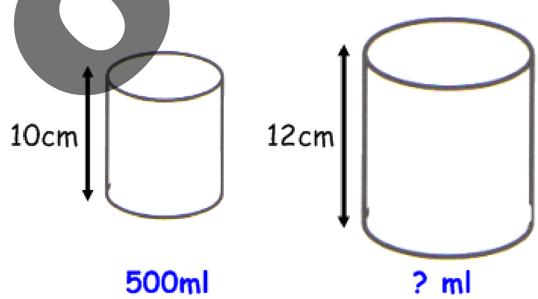
$$\text{s.f.} = \frac{35}{20} = 1\cdot 75$$

Step two: calculate the area, remembering to *square* the scale factor for area.

$$\text{Area} = 2500 \times 1\cdot 75^2 = \underline{\underline{7656\cdot 25\text{cm}^2}}$$

Example 2 – volume

Two cylindrical drinks cans are mathematically similar. The smaller can holds 500ml of juice. Calculate the volume of the larger can.



Solution

Step one: calculate the scale factor.

$$\text{s.f.} = \frac{12}{10} = 1\cdot 2$$

Step two: calculate the volume, remembering to *cube* the scale factor for volume.

$$\text{Area} = 500 \times 1\cdot 2^3 = \underline{\underline{864\text{ml}}}$$

Example 3 – backwards

Two rectangles are mathematically similar. The area of the larger rectangle is double the area of the smaller rectangle. If the breadth of the smaller rectangle is 10cm, find the breadth of the larger rectangle.

Solution

The scale factor connecting the areas is s^2. The information in the question tells us that $s^2 = 2$. Solving this equation tells us that s must be $\sqrt{2}$.

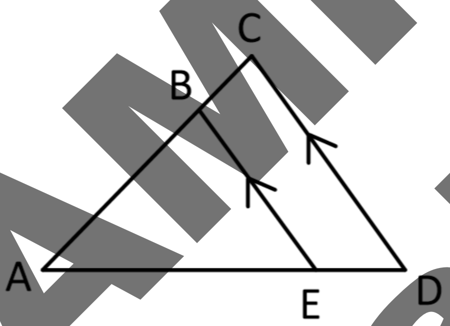
Therefore, the breadth of the larger rectangle is $10 \times \sqrt{2} = \underline{\underline{14 \cdot 1\text{cm}}}$ (1 d.p.)

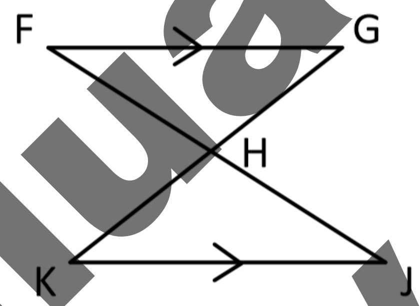
Similarity can also be used to work out lengths in diagrams involving triangles.

Fact: if two triangles have the exact same angles, those two triangles are similar.

This means that triangles in diagrams involving parallel lines, or where triangles share sides, may involve similarity. For example, in the diagrams below:

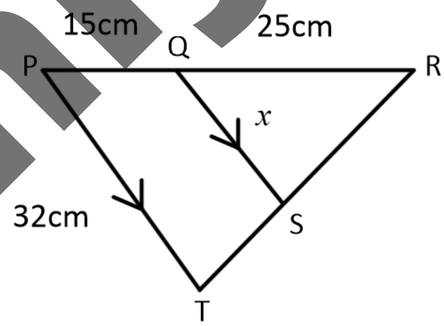
- In the diagram on the left, triangles ABE and ACD are similar.
- In the diagram on the right, triangles FGH and KHJ are similar.





Example 4 – similar triangles

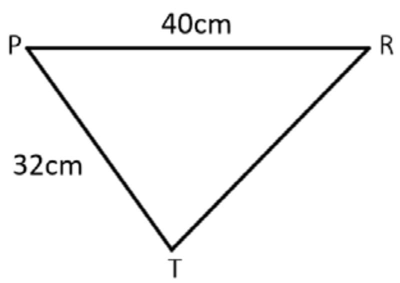
Triangles PRT and QRS are similar as shown.
PQ = 15cm, QR = 25cm and PT = 28cm.
Calculate the length, *x*, of QS.

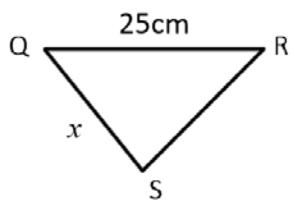


Solution

Firstly, notice that PR = 15 + 25 = 40cm.

Next, it will be less confusing if we redraw the diagram so that the two triangles can be seen side by side:





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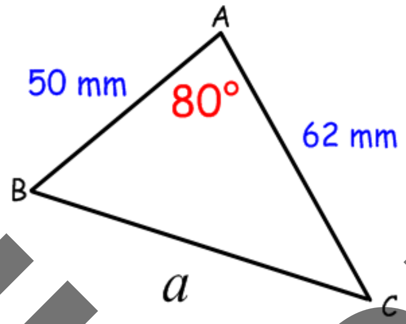
Example 1

Calculate the length *a* in triangle ABC.



Solution

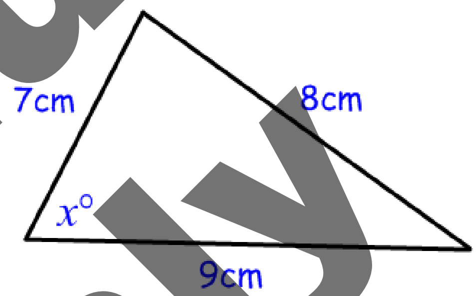
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 50^2 + 62^2 - 2\times 50\times 62\times \cos 80$$
$$= 6344 - 6200\cos 80$$
$$= 5267\cdot 3881298...$$
$$a = \sqrt{5267\cdot 388...}$$
$$= 72\cdot 5767...$$
$$= \underline{72\cdot 6\text{mm}} \text{ (1 d.p.)}$$

To calculate an **angle** using the cosine rule, you must know the lengths of all three sides to be able to use this formula. To find an angle, you use the second version of the formula.

In the formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, it is crucial that *a* must be the side opposite the angle you are finding. It does not matter which way around *b* and *c* go.

Example 2 – finding an angle

Calculate the size of angle *x*° in this triangle.



Solution

Length '*a*' must be the side opposite the angle we are finding, so *a* = 8.

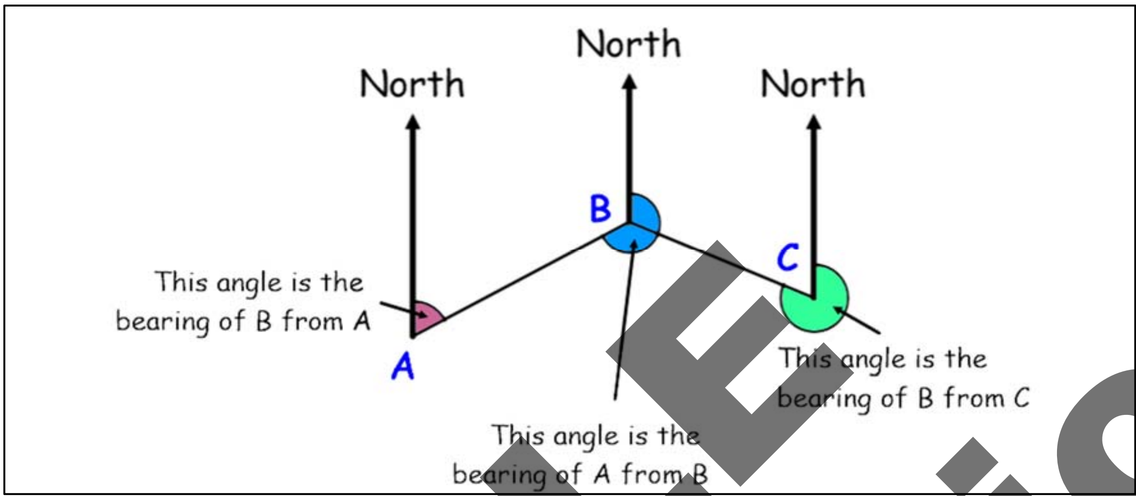
It does not matter which way around *b* and *c* go, so we will say *b* = 7 and *c* = 9:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos x = \frac{7^2 + 9^2 - 8^2}{2\times 7\times 9}$$
$$= \frac{66}{126}$$
$$x = \cos^{-1}\left(\tfrac{66}{126}\right)$$
$$= 58\cdot 411864...$$
$$= \underline{58\cdot 4^\circ} \text{ (1 d.p.)}$$

Bearings

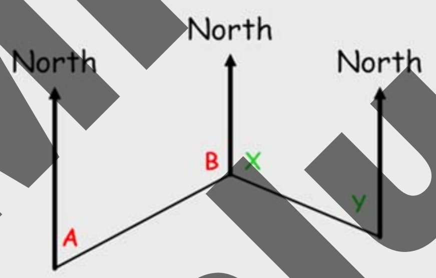
You also need to be confident with handling bearings in diagrams. The bearing is the angle measured clockwise from North (with North being 000°).

When describing bearings, it is important to go the correct way. The bearing of A **from** B refers to the angle that is centred at B and pointing towards A, as demonstrated in the diagram on the next page.



Fact

In a diagram involving bearings, some angles add to make 180°. In the diagram below, A and B add to make 180°, and so do X and Y.

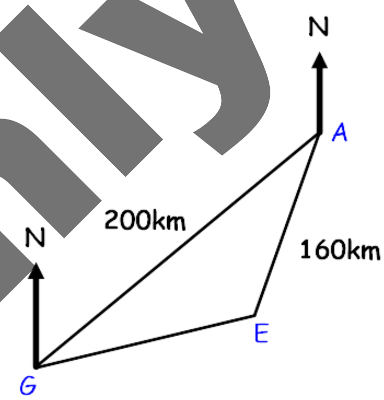


Example (past SQA Standard Grade Credit question)

The diagram shows the position of airports A, E and G.

- **G is 200 kilometres from A.**
- **E is 160 kilometres from A.**
- **From G the bearing of A is 052°.**
- **From A the bearing of E is 216°.**

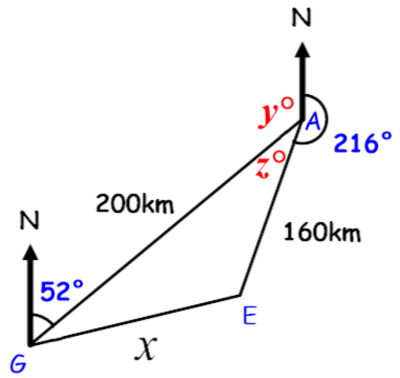
Calculate the distance between airports G and E.



Solution

Step one: put the information from the question into the diagram

We begin by adding the angles to the diagram. Also since the distance we are being asked to find is GE, we can label that side as x.



In real life, resultant vectors can be used to work out what the combined effect of more than one force pulling on an object will be.

Example 1 – numerical

Three forces act on an object. The three forces are represented by the vectors a, b and c, where:

$$\mathbf{a}=\begin{pmatrix}-1\\3\\2\end{pmatrix}\qquad \mathbf{b}=\begin{pmatrix}0\\-5\\6\end{pmatrix}\qquad \mathbf{c}=\begin{pmatrix}4\\0\\2\end{pmatrix}$$

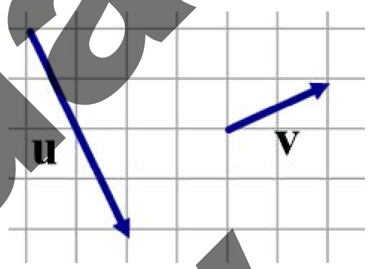
Calculate the resultant force. Express your answer in component form.

Solution

The resultant force is given by $\mathbf{a}+\mathbf{b}+\mathbf{c}=\begin{pmatrix}(-1)+0+4\\3+(-5)+0\\2+6+2\end{pmatrix}=\begin{pmatrix}3\\-2\\10\end{pmatrix}$.

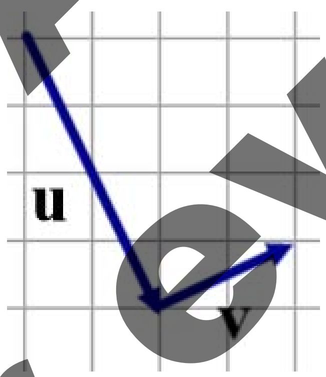
Example 2 – from a diagram

The diagram on the right shows two directed line segments u and v. Draw the resultant vector u + v.

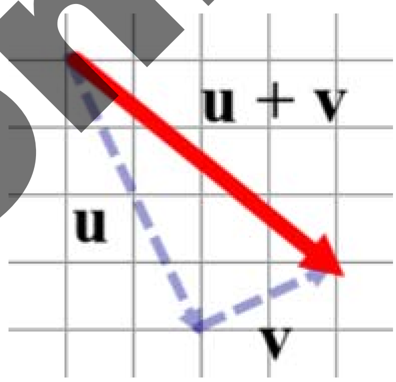


Solution

To add the vectors, we join the 'tail' of **v** to the 'nose' (pointed end) of **u**:



We can now draw in the vector **u** + **v** going from the 'nose' of **u** to the 'tail' of **v**.



Vector Pathways

We can use the rules of adding and taking away vectors to express a vector $\overrightarrow{AB}$ in a diagram as a combination of other, known, vectors.

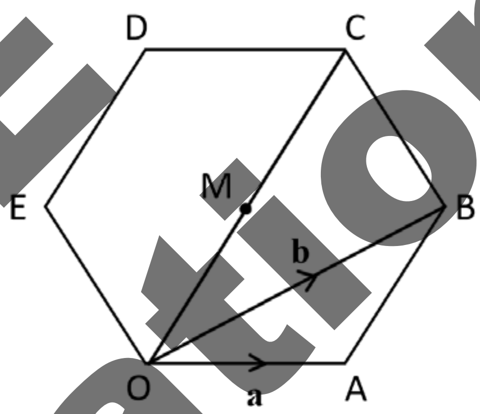
To do this, we identify a route, or path, between A and B, in which each step of the route can be expressed in terms of one of the other known vectors. We can choose *any* route we like, and the final answer, when simplified, will always be the same.

Fact: If we move backwards along a vector, we take that vector away.

Example – vector pathways (adapted from SQA specimen paper)

In the diagram, OABCDE is a regular hexagon with centre M. Vectors a and b are represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively.

Express $\overrightarrow{AB}$ and $\overrightarrow{OC}$ in terms of a and b.



Solution

For $\overrightarrow{AB}$:

Step one: identify a path from A to B.

One possible path is $\overrightarrow{AO}$, $\overrightarrow{OB}$.

Step two: express each part of the path in terms of a known vector:

$\overrightarrow{AO} =$ backwards along $\mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

Therefore $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ (or $\mathbf{b} - \mathbf{a}$).

For $\overrightarrow{OC}$:

Step one: identify a path from O to C.

One possible path is $\overrightarrow{OM}$, $\overrightarrow{MC}$.

Step two: express each part of the path in terms of a known vector:

$\overrightarrow{OM}$ and $\overrightarrow{MC}$ are both the same as $\overrightarrow{AB}$ which is $\mathbf{b} - \mathbf{a}$ (from part (a))

Therefore $\overrightarrow{OC} = 2(\mathbf{b} - \mathbf{a})$ or $2\mathbf{b} - 2\mathbf{a}$.

Multiplying a Vector by a Scalar

A **scalar** is a quantity that has size but no direction. 'Normal' numbers such as 2, –5 or 14·1 are scalars.

We can multiply a vector by a scalar in two ways:

- **Numerically** by multiplying each component of the vector.

If we have a vectors, $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and a scalar k, then $k\mathbf{a} = \begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$.

Statistics

Scatter Graphs and Line of Best Fit

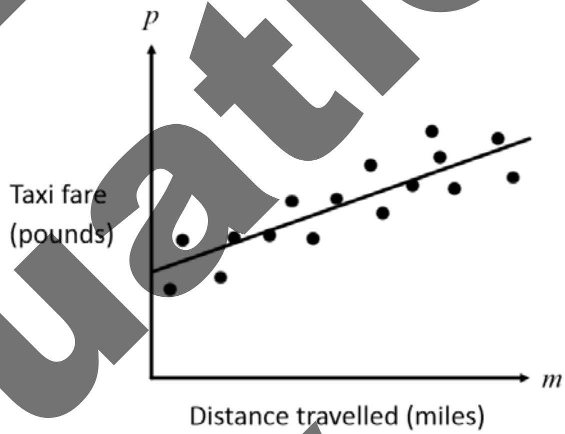
The line of best fit on a scatter graph is a straight line. This means that you can find the equation of a line of best fit using the method ($y-b=m(x-a)$) outlined on page 45.

Once you have the equation, you can use the equation to estimate the value of *y* when you are told *x* (or vice versa). <u>At National 5, **you must use the equation**</u> to get any marks (the question will state this). You *<u>cannot</u>* do it by "looking and guessing". Any answer without working will get zero marks, even if it happens to be correct.

<u>Example 1</u> (2010 Intermediate 2 Exam Question)

A scatter graph shows taxi fare (*p* pounds) plotted against the distance travelled, (*m* miles). A line of best fit has been drawn.

The equation of the line of best fit is $p=2+1\cdot5m$. Use this equation to predict the taxi fare for a journey of 6 miles.



Solution

The journey is 6 miles, so m = 6. Using the equation:

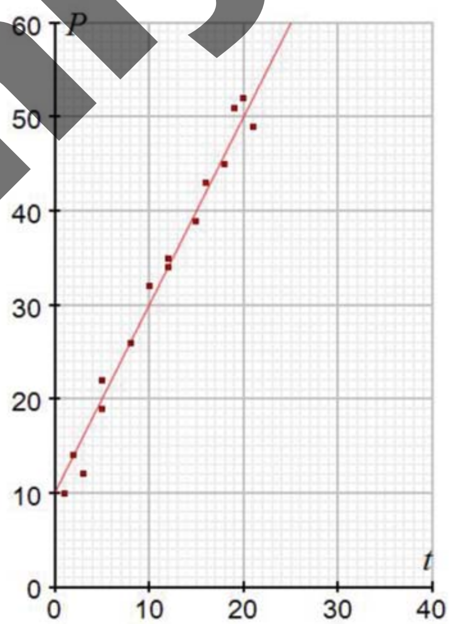
$$p=2+1\cdot5m$$
$$p=2+1\cdot5\times6$$
$$p=\underline{11\text{ miles}}$$

<u>Example 2</u>

The scatter graph shows the power of an industrial battery (*P*) after *t* hours of charging.

(a) Find the equation of the line of best fit in terms of *P* and *t*. Give the equation in its simplest form.

(b) Use your equation to estimate the power of a battery that has been charged for 60 hours.



Solution

(a) We use the usual method for $y-b=m(x-a)$ (see page 45). On this occasion, since the letters *P* and *t* are being used, we will adapt the formula to be $P-b=m(t-a)$.

The *y* intercept is 10, so *c* = 10.

Now choose two points **<u>on the line of best fit</u>** (not necessarily on the original scatter graph) to calculate the gradient.

(continued on next page)

Comparing Statistics

The **mean**, **median** and **mode** are averages. They tell us whether a list of numbers is higher or lower on average. You should include the words "on average" in a comment.

The **range**, **semi-interquartile range** and **standard deviation** are measures of spread. They tell us whether a list of numbers is more or less varied/consistent.

- A lower range, SIQR or standard deviation means the numbers are more **consistent**.
- A higher range, SIQR or standard deviation means the numbers are more **varied**.

Example

The temperature in Aberdeen has a mean of 3°C and a standard deviation of 5. In London it has a mean of 9°C and a standard deviation of 3.
Make two comments comparing the temperatures in London and Aberdeen.

Solution

You would get NO MARKS (because you are stating the obvious) for:

- *"Aberdeen has a lower mean".*
- *"London has a higher mean".*
- *"Aberdeen has a higher standard deviation".*
- *"London has a lower standard deviation".*

You would get NO MARKS (because your sentence makes no sense) for:

- *"Aberdeen is lower".* (no mention of temperature)
- *"The first one has a lower temperature".* (no mention of Aberdeen or London)
- *"In London it is more consistent".* (no mention of what 'it' is)
- *"The standard deviation in London is more consistent".* (it is the temperature that is more consistent, not the standard deviation).

You would get NO MARKS (because your sentence makes no sense) for:

- "Temperatures in Aberdeen are lower" because temperatures in Aberdeen will not always be lower. However, you would get a mark for "On average, temperatures in Aberdeen are lower than London" or "Temperatures in Aberdeen are usually lower than London".

You WOULD get marks for sentences such as:

- *"On average, the temperature in Aberdeen is lower than London and the temperature is less consistent"*
- *"The temperature in London is higher and more consistent than Aberdeen"*

Note: you do *not* need two numbers and a comparing word in this type of sentence.

Index of Key Words