

## Index of Key Words

|  |                                  |   |                                  |
|--|----------------------------------|---|----------------------------------|
| ! notation.....                        | 65                               | Squaring .....                                | 159                              |
| ■ (symbol for end of a proof) .....    | 9, 178                           | Complex Plane .....                           | 157, 165                         |
| Absolute Value .....                   | 107                              | Conjecture .....                              | 174                              |
| Acceleration .....                     | 116, 118                         | Conjugate.....                                | <i>See</i> Complex Numbers       |
| Algebraic Long Division .....          | 11                               | Conjugate Roots Property .....                | 72                               |
| Algorithm                              |                                  | Continuous.....                               | 100                              |
| Algebraic Long Division.....           | 11                               | Contrapositive .....                          | 176                              |
| Division .....                         | 169                              | Proof.....                                    | 182                              |
| Euclidean .....                        | 169                              | Converse .....                                | 176                              |
| Angles in 3-d.....                     | 148                              | cosec .....                                   | 19                               |
| Angle Between a Line and a Plane.....  | 149                              | cosecant.....                                 | <i>See</i> cosec                 |
| Angle Between Two Lines.....           | 149                              | cot .....                                     | 19                               |
| Angle Between Two Planes.....          | 148                              | cotangent.....                                | <i>See</i> cot                   |
| Argand Diagram .....                   | 157                              | Counterexample .....                          | 176                              |
| Argument .....                         | 157                              | Cross Product.....                            | 141                              |
| Arithmetic Sequences .....             | 78                               | Cubic Equation.....                           | 72                               |
| Asymptote .....                        | 100                              | Curve Sketching .....                         | 102                              |
| Auxiliary equation .....               | 56, 58, 59                       | $f(x)$   .....                                | 107                              |
| Bases.....                             | <i>See</i> Changing Number Bases | Inverse Functions .....                       | 105                              |
| Binary .....                           | <i>See</i> Changing Number Bases | Rational Functions.....                       | 102                              |
| Binomial Coefficients .....            | 65                               | de Moivre's Theorem .....                     | 160                              |
| Binomial Theorem.....                  | 67                               | Formulae involving Powers of sin and cos..... | 163                              |
| Specific Terms.....                    | 68                               | Identities .....                              | 163                              |
| Brackets.....                          | <i>See</i> Binomial Theorem      | Proof by Induction.....                       | 96                               |
| Cartesian Form                         |                                  | Proof of .....                                | 96                               |
| Complex Numbers .....                  | 157                              | Roots .....                                   | 160                              |
| Plane (Equation of) .....              | 146                              | Decimal .....                                 | <i>See</i> Changing Number Bases |
| Chain Rule .....                       | 21                               | Determinant .....                             | 133                              |
| Three or More Functions .....          | 22                               | Differentiable.....                           | 110                              |
| Changing Number Bases .....            | 172                              | Differential Equations.....                   | 50                               |
| Command Words .....                    | 8                                | First Order Linear .....                      | 52                               |
| Common Difference .....                | 78                               | Integrating Factor Method.....                | 52                               |
| Common Ratio.....                      | 81                               | Second Order Linear.....                      | 55                               |
| Complementary Function.....            | 60                               | Second Order Linear Homogeneous .....         | 56                               |
| Complex numbers                        |                                  | Second Order Linear Non-Homogeneous ..        | 60                               |
| Add and Take Away .....                | 71                               | Separable .....                               | 51                               |
| Dividing.....                          | 159                              | Differentiation                               |                                  |
| Equating Real and Imaginary Parts..... | 75                               | Chain Rule .....                              | 21                               |
| Multiplying.....                       | 71, 159                          | Differentiable Functions.....                 | 110                              |
| Real and Imaginary Parts .....         | 70                               | Higher Derivatives .....                      | 20                               |
| Solving Equations.....                 | 72                               | Implicit .....                                | 26                               |
| Squaring.....                          | 71                               | Logarithmic .....                             | 30                               |
| Complex Numbers.....                   | 70, 157                          | Parametric.....                               | 31                               |
| Argument.....                          | 157                              | Planning How to Answer a Question....         | 24, 26                           |
| Conjugate.....                         | 71, 75                           | Product Rule.....                             | 22                               |
| de Moivre's Theorem .....              | 160                              | Quotient Rule .....                           | 23                               |
| Dividing.....                          | 72                               | Rational Functions.....                       | 23                               |
| Equation.....                          | 75                               | Second Derivative .....                       | 20                               |
| Fractions .....                        | 72                               | Stationary Points .....                       | 109                              |
| Modulus.....                           | 157                              | Using More Than One Rule .....                | 24                               |
| Powers .....                           | 160                              | Dilatation .....                              | 139                              |
| Quadratic Equations .....              | 70                               | Discontinuous .....                           | 100, 110                         |
| Roots.....                             | 160                              | Displacement .....                            | 116, 118                         |
| Square Roots.....                      | 76, 161                          | Disprove .....                                | 176                              |

|   |                      |
|---|----------------------|
| Division .....                                | 11, 169              |
| Division Algorithm .....                      | 169                  |
| Domain .....                                  | 98                   |
| Dummy Variable.....                           | 21, 46               |
| Elementary Row Operations .....               | 121                  |
| Empty Set .....                               | 10                   |
| Enlargement Matrix .....                      | 139                  |
| Equations                                     |                      |
| Cubic .....                                   | 72                   |
| in Complex Plane .....                        | 165                  |
| Quartic .....                                 | 72                   |
| With Complex Roots .....                      | 72                   |
| Equivalence .....                             | 176                  |
| Equivalent.....                               | 177                  |
| Euclidean Algorithm .....                     | 169                  |
| Backwards.....                                | 170                  |
| Even  |                      |
| Functions .....                               | 98                   |
| Numbers .....                                 | 174                  |
| Exact Values of sin, cos and tan              |                      |
| in radians .....                              | 7                    |
| Expand Brackets .....                         | See Binomial Theorem |
| Express                                       |                      |
| As Sum of Polynomial and Proper Rational      |                      |
| Function .....                                | 11                   |
| Extreme Values.....                           | 111                  |
| Factor .....                                  | 178                  |
| Factorial (!) .....                           | 65                   |
| For All ( $\forall$ ).....                    | 175                  |
| Formulae .....                                | 3                    |
| Angle between Two Vectors.....                | 148                  |
| Area and Volume .....                         | 7                    |
| Arithmetic Sequences.....                     | 79                   |
| Binomial Coefficient .....                    | 66                   |
| Binomial Theorem .....                        | 67                   |
| Binomial Theorem General Term .....           | 68                   |
| Derivatives .....                             | 20, 21               |
| Determinant of a $2 \times 2$ Matrix.....     | 133                  |
| Equation of a line in 3d.....                 | 146                  |
| Equation of a Line in 3d .....                | 143                  |
| Geometric Sequence .....                      | 81                   |
| Integrals.....                                | 35, 36, 41, 52, 53   |
| Integration by Parts .....                    | 46                   |
| Inverse of a $2 \times 2$ Matrix.....         | 137                  |
| Maclaurin Series .....                        | 85                   |
| Maclaurin Series ( $e$ , sin, cos, ...) ..... | 86                   |
| Particular Integral .....                     | 60                   |
| Product Rule .....                            | 22, 32               |
| Quotient Rule .....                           | 23                   |
| Rectilinear Motion .....                      | 116                  |
| Scalar Triple Product.....                    | 142                  |
| sec, cosec, cot.....                          | 19                   |
| Sum of $r$ , $r^2$ , $r^3$ .....              | 90                   |
| Sum to Infinity .....                         | 84                   |
| Transformation Matrices.....                  | 139                  |
| Trigonometry .....                            | 19                   |
| Vectors (basic) .....                         | 141                  |
| Volume of Solid of Revolution .....           | 114                  |
| Fractions                                     |                      |
| Simplifying (Numerical).....                  | 8, 9                 |
| Functions .....                               | 98                   |
| Fundamental Theorem of Algebra.....           | 72                   |
| Fundamental Theorem of Arithmetic....         | 180                  |
| Gaussian Elimination .....                    | 121, 137, 155        |
| Ill-Conditioning.....                         | 127                  |
| Inconsistent Equations.....                   | 125                  |
| Redundant Equations.....                      | 124                  |
| General Solution See Differential Equations   |                      |
| Geometric Series .....                        | 81                   |
| Sum to Infinity.....                          | 84                   |
| Geometrically.....                            | 150, 165             |
| Graphs  |                      |
| $ f(x) $ .....                                | 107                  |
| Inverse Functions .....                       | 105                  |
| Odd and Even Functions .....                  | 98                   |
| Greatest Common Divisor (gcd) .....           | 169                  |
| Greek Letters .....                           | 10                   |
| Hexadecimal .. See Changing Number Bases      |                      |
| Highest Common Factor (hcf).....              | 169                  |
| Homogeneous Differential Equations .....      | 56                   |
| $i$ (number) .....                            | 70                   |
| Identity Matrix.....                          | 130, 132             |
| If and only If .....                          | 177                  |
| Iff See If and Only If                        |                      |
| Ill-Conditioning .....                        | 127                  |
| Imaginary Number .....                        | 70                   |
| Imaginary Part .....                          | 70                   |
| Implicit Differentiation .....                | 26                   |
| Second Derivative .....                       | 29                   |
| Implies/Implied by (proof).....               | 177                  |
| Improper Fractions .....                      | 11                   |
| Inconsistency .....                           | 125, 155             |
| Independent of $x$ .....                      | 69                   |
| Induction (Proof).....                        | 92                   |
| Inequations                                   |                      |
| in Complex Plane.....                         | 165                  |
| Infinite Geometric Series .....               | 84                   |
| Inflection Point .....                        | 109                  |
| Integrating Factor .....                      | 52                   |
| Integration                                   |                      |
| Areas with Respect to $y$ .....               | 113                  |
| By Parts .....                                | 45                   |
| by Substitution .....                         | 36                   |
| Partial Fractions .....                       | 43                   |
| Rational Functions.....                       | 43                   |
| Solids of Revolution.....                     | 114                  |
| Volumes .....                                 | 114                  |
| Intersections in 3-d .....                    | 150                  |
| a Line and a Plane .....                      | 151                  |
| Three Planes.....                             | 156                  |
| Two Lines.....                                | 151                  |
| Two Planes .....                              | 154                  |
| Inverse                                       |                      |
| Functions.....                                | 105                  |
| Matrices .....                                | 135                  |

|  |                             |                                    |                  |
|--|-----------------------------|------------------------------------|------------------|
| Irrational.....                        | 174                         | Partial Fractions .....            | 13, 43           |
| Proof .....                            | 181                         | Basic method.....                  | 14               |
| Irreducible Quadratic Factor .....     | 16                          | Partial Fractions                  |                  |
| Lambda ( $\lambda$ ) .....             | 10                          | Repeated Linear Factor .....       | 15               |
| Lines in 3-d .....                     | 143                         | Partial Fractions                  |                  |
| Angle Between.....                     | 149                         | Improper Fractions.....            | 18               |
| Equation.....                          | 143                         | Particular Integral .....          | 60               |
| Intersection.....                      | 151                         | Particular Solution .....          | See Differential |
| Skew.....                              | 150                         | Equations                          |                  |
| Locus.....                             | 165                         | Parts (Integration by).....        | 45               |
| Logarithmic Differentiation .....      | 30                          | Pascal's Triangle.....             | 65               |
| Logic .....                            | 174                         | Planes in 3-d .....                | 146              |
| Long Division ....                     | See Algebraic Long Division | Angle Between .....                | 148              |
| Maclaurin Expansion .....              | 85                          | Equation .....                     | 146              |
| Maclaurin Series.....                  | 85                          | Intersection .....                 | 154, 156         |
| Matrices.....                          | 121, 128                    | Point of Inflection .....          | 109              |
| Add and Subtract .....                 | 129                         | Polar Form .....                   | 157              |
| Determinant .....                      | 133                         | Prime Numbers.....                 | 169              |
| Element.....                           | 128                         | Proof of Infinity of Primes .....  | 181              |
| Identity Matrix.....                   | 130                         | Product Rule .....                 | 22               |
| Inverse .....                          | 135                         | Proof .....                        | 92, 174          |
| Multiplication .....                   | 130                         | By Contradiction.....              | 179              |
| Multiply by a Scalar.....              | 129                         | By Contrapositive .....            | 182              |
| Order.....                             | 128                         | By Induction .....                 | 92               |
| Proof by Induction .....               | 95                          | Direct Proof .....                 | 177              |
| Squaring.....                          | 131                         | Disproving by Counterexample ..... | 176              |
| Symmetric.....                         | 130                         | Factor .....                       | 178              |
| Transformation.....                    | 139                         | Infinity of Primes .....           | 182              |
| Transpose .....                        | 129                         | Of Irrationality.....              | 181              |
| Maximum and Minimum Values ....        | 111, 119                    | Proof by Induction                 |                  |
| Modulus .....                          | 107, 157                    | Complex Numbers.....               | 96               |
| Motion                                 |                             | Differentiation.....               | 97               |
| 2 dimensions.....                      | 118                         | Divisibility .....                 | 94               |
| Rectilinear.....                       | 116                         | Matrices .....                     | 95               |
| Straight Line.....                     | 116                         | Series.....                        | 93               |
| Multiple .....                         | 178                         | Proper Fractions .....             | 11               |
| Natural Numbers.....                   | 10, 174                     | Quartic Equation.....              | 72               |
| Nature Table.....                      | 109                         | Quotient Rule.....                 | 23               |
| Negation .....                         | 174, 176                    | $\mathbb{Q}$ (symbol).....         | 10               |
| Non-Homogeneous Differential Equations |                             | Range .....                        | 98               |
| .....                                  | 60                          | Rate.....                          | 33, 63           |
| Non-Singular.....                      | 135                         | Ratio (Geometric Series).....      | 81               |
| Normal Vector .....                    | 146                         | Rational.....                      | 174              |
| $n$ th Roots .....                     | 160                         | Functions.....                     | 43               |
| Null Set .....                         | 10                          | Functions (Graph Sketching) .....  | 100              |
| $\mathbb{N}$ (symbol) .....            | 10                          | Numbers.....                       | 10               |
| Odd                                    |                             | Real Numbers .....                 | 10               |
| Functions .....                        | 98                          | Real Part.....                     | 70               |
| Numbers .....                          | 174                         | Rearranging.....                   | 50               |
| Only if (proof).....                   | 177                         | Rectilinear Motion .....           | 116              |
| Optimisation.....                      | 111, 119                    | Redundancy .....                   | 124, 155         |
| Orthogonal .....                       | 136                         | Reflection Matrix .....            | 139              |
| Parameter.....                         | 31, 124, 144, 154           | Related Rates of Change.....       | 33               |
| Parametric Equations .....             | 31, 118                     | Relatively Prime .....             | 169              |
| Differentiation .....                  | 31                          | Rotation Matrix.....               | 139              |
| Line in 3-d .....                      | 143, 146                    | $\mathbb{R}$ (symbol).....         | 10               |

|   |                |  |          |
|---|----------------|--|----------|
| Scalar Triple Product .....                         | 142            | Symmetric Form .....                         | 143      |
| sec .....   | 19             | Systems of Equations.....                    | 121      |
| secant .....  | <i>See</i> sec | There Exists ( $\exists$ ) .....             | 175      |
| Second Derivative Test.....                         | 109            | Theta ( $\theta$ ) .....                     | 10       |
| Second Order Linear Differential Equations<br>..... | 55             | Transformation Matrices .....                | 139      |
| Separable Differential Equations .....              | 51             | Transpose.....                               | 129      |
| Sequences .....                                     | 78             | Trigonometric Identities .....               | 19       |
| Sequences and Series                                |                | Trigonometry                                 |          |
| Arithmetic.....                                     | 78             | Exact Values .....                           | 7        |
| Geometric.....                                      | 81             | Upper Triangular Form .....                  | 121      |
| Maclaurin.....                                      | 85             | Vector Form (Equation)                       |          |
| Series.....   | 78             | Line in 3d.....                              | 143      |
| Sets.....   | 10             | Plane in 3d.....                             | 146      |
| Show That.....                                      | 9              | Vector Product..... <i>See</i> Cross Product |          |
| Sigma Notation.....                                 | 78             | Vectors.....                                 | 141      |
| Singular.....                                       | 134, 135       | Cross Product .....                          | 141      |
| Sketching Rational Functions .....                  | 102            | Normal .....                                 | 146      |
| Skew Lines .....                                    | 150, 151       | Scalar Triple Product .....                  | 142      |
| Stationary Points .....                             | 109            | Velocity .....                               | 116, 118 |
| Substitution (Integration).....                     | 36             | Volume.....                                  | 114      |
| Sum to Infinity .....                               | 84             | $\mathbb{Z}$ (symbol).....                   | 10       |
| Summation Formulae.....                             | 90             |  |          |