

The negation of p (known as **not p or $\neg p$**): not p : $2 + 3 \geq 7$ not p is **false**

The negation of q (known as **not q or $\neg q$**): not q : 46 is not a prime number not q is **true**

Some statements are **universal**, meaning they are true for all values of x . ('for all x ', written $\forall x$). A universal statement is harder to prove as it must be checked for every value of x .

Other statements are statements of **existence**, meaning they are true for at least one value of x . ('there exists an x ', written $\exists x$). An existence statement is easier to prove as it is enough just to find one example that works.

The negation of a universal statement is an existence statement and vice versa.

Examples 2

If statement a is "all grass is green", then **not a** is "some grass is not green"

If statement b is " $x^2 + 1$ is odd $\forall x$ ", then **not b** is " $\exists x$ such that $x^2 + 1$ is even"

Some statements are of the form 'A and B' or 'A or B'

- the negation of 'A and B' is 'not (A and B)' which is the same as '(not A) or (not B)'
In symbols: $\neg(A \text{ and } B) = \neg A \text{ or } \neg B$
- the negation of 'A or B' is '(not A) and (not B)'. In symbols:
 $\neg(A \text{ or } B) = \neg A \text{ and } \neg B$

Example 3

Given that m and n are positive integers, state the negation of the statement ' m is prime and n is prime'

Solution

Let statement A be ' m is prime'.

Let statement B be ' n is prime'.

We need the negation of $\neg(A \text{ and } B)$, which is $\neg A$ or $\neg B$.

The negation is ' m is not prime or n is not prime'.

Some statements are of the form 'if A then B'. In mathematical notation this can be written $A \Rightarrow B$.

- The **contrapositive** of $A \Rightarrow B$ is $\text{not } B \Rightarrow \text{not } A$. If the original statement is true (or false) then the contrapositive must also be true or false. Sometimes it can be easier to prove the contrapositive than the original statement.
- The **negation** of $A \Rightarrow B$ is 'there exists something that is both not B and A '.

Example 4

A statement is 'if something is a dinosaur then it is dead'. State the contrapositive and the negation.

Solution

The original statement can be thought of as $A \Rightarrow B$, where A is 'being a dinosaur' and B is 'being dead'.

The **contrapositive** ($\text{not } B \Rightarrow \text{not } A$) could be written '*if something is not dead then it is not a dinosaur*'.