

Formula (this is not given on the formula sheet)

To solve a first order separable differential equation, use the formula $I(x)y = \int I(x)Q(x)dx$.

Example 1 – no initial conditions

Solve $\frac{dy}{dx} + y = 6e^{2x}$. **Express your answer in the form** $y = f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

On this occasion, it is already in this form, so we do not need to rearrange.

Step two: identify $P(x)$ and $Q(x)$

$$P(x) = 1 \text{ and } Q(x) = 6e^{2x}.$$

Step three: find the integrating factor using the formula $I(x) = e^{\int P(x)dx}$.

$$\begin{aligned} I(x) &= e^{\int P(x)dx} \\ &= e^{\int 1dx} = e^x \end{aligned}$$

Step four: use the formula $I(x)y = \int I(x)Q(x)dx$ to find the general solution, and rearrange to make y the subject.

$$\begin{aligned} I(x)y &= \int I(x)Q(x) dx \\ e^x y &= \int e^x \cdot 6e^{2x} dx \\ e^x y &= 6 \int e^{3x} dx \\ e^x y &= 6 \cdot \frac{1}{3} e^{3x} + C \\ y &= \underline{\underline{2e^{2x} + \frac{C}{e^x}}} \end{aligned}$$

Sometimes the equation must be rearranged into the standard form before we can solve it.

Example 2 – requires rearranging

Solve $x \frac{dy}{dx} - 2y - x^3 \sin x = 0$. **Express your answer in the form** $y = f(x)$.

Solution

Step one: rearrange into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

$$x \frac{dy}{dx} - 2y - x^3 \sin x = 0$$

$$x \frac{dy}{dx} - 2y = x^3 \sin x \quad (\text{move } x^3 \sin x \text{ to the other side})$$

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \sin x \quad (\text{divide through by } x)$$

(continued on next page)