National 5 Mathematics
Revision Notes

Last updated August 201[illegible]

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Use this booklet to practise working independently l[illegible] will [illegible] to in the exam.
- Get in the habit of turning to this booklet to refresh [illegible]ry.
- If you have forgotten how to do a me[illegible]d, **examples** a[illegible]en.
- If you have forgotten what a word m[illegible]se the **i**[illegible]**x** (back page) to look it [illegible]

As you get closer to the final tes[illegible]uld [illegible]g to use this bookle[illegible]d le[illegible]

This booklet is [illegible]r:
- Students [illegible] N[illegible]na[illegible]athematics course.
- Students s[illegible]g [illegible]ore of the National 5 Mathem[illegible]nits: **Expressions and Formulae**, **R**[illegible] or **Application**[illegible]

Thi[illegible]**ains:**
- T[illegible]os[illegible]nt facts you need to m[illegible] [illegible]ational 5 Mathematics.
- Ex[illegible]hat take you through the [illegible]mo[illegible]**utine** questions in each [illegible].
- Def[illegible]ns of the key words you [illegible]

[illegible]s booklet:
- [illegible] refresh your memory of th[illegible]ethod you were taught in class whe[illegible] are stu[illegible] a homework question o[illegible]st question.
- To memorise key [illegible] whe[illegible]g for the exam.

The key to revisin[illegible]r [illegible]at[illegible] exam is to do questions, not to [illegible]otes. **As well as using this booklet, you** [illegible]**d a**[illegible]
- Revise by work[illegible]ugh exercises on topic[illegible]u ne[illegible]ore practice on – such as revi[illegible]n booklets, textbooks, websites, or othe[illegible]rcis[illegible]uggested by your teacher.
- W[illegible] through practice tests.
- A[illegible]ur teacher when you come across a question you cannot answer.
- Use [illegible]rces online (a link that can be scanned with a SmartPhone is on the last page)

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Percentages ..... [illegible]

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All i[illegible]ation in t[illegible]vision guide has bee[illegible]pared [illegible]aith, with thorough reference to [illegible]s provided by the SQA, includ[illegible]e cou[illegible]rrangements, course and unit [illegible]por[illegible] exam specification, specimen [illegible]ion paper and unit assessm[illegible]nts.

[illegible]se notes will be updated a[illegible]nd [illegible]w information become a[illegible]ilab[illegible]

[illegible]y our hardest to ensure th[illegible] note[illegible] curate, but despite o[illegible]t efforts, [illegible]kes [illegible]sometimes appear. If you d[illegible]ver any mistakes in these notes, p[illegible] email us [illegible]vid@dynamicmaths.c[illegible] A[illegible]rected replacement copy o[illegible] note[illegible] be p[illegible]ided free of charge! We would als[illegible] hear of any suggestion[illegible] fo[illegible]mprovement.

This version is ve[illegible]2.0[illegible]ublished July 2015

Previou[illegible]versions:
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Ve[illegible]2. January 2014.
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With grateful thanks to **Arthur McLaughlin** and **John Stobo** for their proof reading.

Formula Sheet

The following formulae from these notes are collected on this page for ease of reference:

Formulae that <u>are given</u> on the formula sheet in the exam (or in unit assessments)

Topic	Formula(e)	Page Reference
Volume of a Pyramid	$V=\frac{1}{3}Ah$	See page 36
Volume of a Sphere and Cone	**Sphere:** $V=\frac{4}{3}\pi r^3$ **Co**[illegible] h	See page 37
The Quadratic Formula	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	See page 55
Area of a Triangle	$A=\frac{1}{2}ab\sin C$	See pa[illegible]
Sine rule	$\frac{a}{\sin A}$ [illegible] $\sin C$	[illegible]ge 73
Cosine rule	[illegible] $+c$[illegible] $\cos A$ or $\cos A=\frac{b^2+c^2}{}$[illegible]	[illegible]e page 74
Standard dev[illegible]	[illegible] or $\sqrt{\frac{\sum x^2-(\sum x)^2}{n}}$[illegible]	See page 91

Fo[illegible]**t are n**[illegible]**ven in the exam** (o[illegible]it ass[illegible]s)

Topi[illegible]	Formula(e)	Pag[illegible] Re[illegible]nce
Comple[illegible]he [illegible] (q[illegible]r method)	$p=\frac{b}{2}$ [illegible]	[illegible] 28
Gr[illegible]nt	$m=\frac{y_2}{}$[illegible]	See pag[illegible]
Arc length and Sector Area	[illegible]**length:** [illegible]$\frac{\text{[illegible]gle}}{360}\pi d$ **Sector A**[illegible]$\frac{}{360}$[illegible]	See pages 35/36
Volume of a prism and cylinder	**Pr**[illegible]**:** $V=Ah$ **C**[illegible]h	See pages 36/37
Straight line	[illegible]$-b=m(x-a)$	See page 42
Discri[illegible]nt	b^2-4ac	See page 56
Pyth[illegible]’ Theorem	$a^2+b^2=c^2$	See page 58
[illegible]	$\sin x^\circ=\frac{\text{Opp}}{\text{Hyp}}$ $\cos x^\circ=\frac{\text{Adj}}{\text{Hyp}}$ $\tan x^\circ=\frac{\text{Opp}}{\text{Adj}}$	See page 60
Tri[illegible]ometry	$\tan x=\frac{\sin x}{\cos x}$ $\sin^2 x+\cos^2 x=1$	See page 71
Magnitude of a vector	$\lvert\mathbf{a}\rvert=\sqrt{a_1^2+a_2^2}$ $\lvert\mathbf{a}\rvert=\sqrt{a_1^2+a_2^2+a_3^2}$	See page 82
Percentage increase and decrease	$\frac{\text{increase (or decrease)}}{\text{original amount}}\times 100$	See page 85
Semi Interquartile Range (SIQR)	$\frac{\text{upper quartile}-\text{lower quartile}}{2}$	See page 91

There are two main methods that can be used. It is possible that you will have been taught one of these, or maybe you will have been taught both and will have chosen the one that you prefer. These can be described as **long multiplication** (where the sum is written out in a similar way to the single-digit multiplication sum, but with additional lines) or the **box method**.

In the box method, each number is split up according to its digits (e.g. 58 is split into 50 and 8; 238 is split into 200, 30 and 8) and then a mini 'tables [illegible]uare' is produced. All the answers in the table square are then added together.

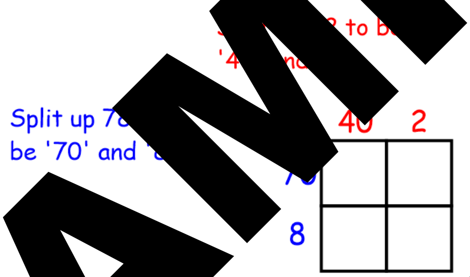
Example 2: Multiplying two two-digit numbers

Multiply 78 × 42

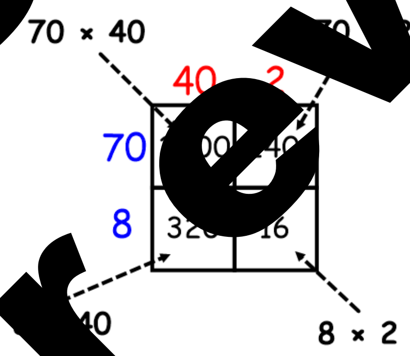
Solutions

Method A (Box Method)

Step 1 – construct a multipli[illegible]on s[illegible]e with two numbers along th[illegible] [illegible]o numbers along the top[illegible]



Step 2 – [illegible]ltiply the numbers in eac[illegible] [illegible]nd [illegible]lumn to obtain one nu[illegible]ber in ea[illegible] of the four smaller squares[illegible]

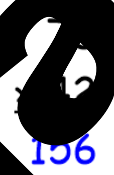


S[illegible] – add the four numbers to obtain the [illegible]al answer.

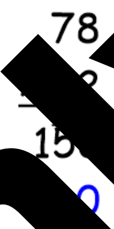
2800 + 140 + 320 + 16 = 3276

Method B (Long [illegible]ultipl[illegible])

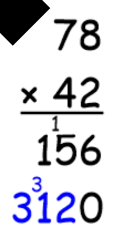
[illegible]p 1 – start doing a us[illegible]ltiplica[illegible] sum and do 78 × 2 using t[illegible] method.



Step [illegible] – [illegible]e n[illegible] line will be for 78 × 4. [illegible]wev[illegible] [illegible]se the sum should really be [illegible]0, we write one zero in the units co[illegible]



Step 3 – c[illegible]te the [illegible]n 78 × 4 usual the usual me[illegible]



Step 4 – add the two answers to obtain the final answer.

156 + 3120 = 3276.

These methods can be extended to any multiplication sum, including multiplication of three- (or more)-digit numbers, or multiplication of decimals.

Example 3: Multiplying three-digit and two-digit numbers

Multiply 247 × 56

Solution

Using **method A** (box method) the working is as follows:

	200	40	7
50	10000	2000	350
6	1200	240	42

10000 + 2000 + 350 [illegible]
+ 1200 + 240 + 42
= 13832

Using **method B** (long multiplication) the working is as follows:

```
  247
 [illegible]56
  2 4
 1482
 2 3
12350
13832
```

The [illegible]al answer is £13832

You might also [illegible]ecte[illegible] multiply decima[illegible] num[illegible] by m[illegible]s of 10, 100 or 1000 (e.g. m[illegible]tiplying by [illegible]r 6000).

The [illegible] to t[illegible]d is splitting into two steps:

- [illegible]nvolves multiplying by a sin[illegible]t usi[illegible] method above.
- [illegible]B involves multiplying by 10[illegible]0 [illegible]0 (by moving all the digits to the le[illegible]

[illegible] no matter which order St[illegible] A an[illegible] a[illegible] done in. The follo[illegible] table [illegible] how this [illegible] be done:

Calculatio[illegible]	Step A	S[illegible]
M[illegible] by 20	Multiply by 2	M[illegible]y by [illegible]
[illegible]ult[illegible]y by 300	Multiply by 3	Mu[illegible] by 100

Example [illegible]: Multiplying by a multiple of 10, 100 o[illegible]00

[illegible]ltiply without a calculator: 3·14 × 3[illegible]

Soluti[illegible]

F[illegible]o 3·14 × 3, and then multiply the answer by 1000, (or 3·14 × 1000, and then × 3).

3·14 × 3 = 9·42 *(working with carrying shown on right)*

9·42 × 1000 = 9420

```
 3·14
× 1 3
 9·42
```

Expressions and Formulae Unit

Surds and Indices

Simplifying Surds

Definition: a **surd** is a square root (or cube root etc.) wh[illegible] does not have an exact answer. e.g. $\sqrt{2} = 1.414213562...$, so $\sqrt{2}$ is a surd. Howe[illegible] $\sqrt{9}$ [illegible] and $\sqrt[3]{64} = 4$, so $\sqrt{9}$ and $\sqrt[3]{64}$ are not surds because they have an exact answ[illegible]

We can multiply and divide surds.

Facts

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\quad$ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ $\quad$ $\sqrt{x} \times \sqrt{x} = (\sqrt{x})^2 = x$

Example 1

Simplify $3\sqrt{2} \times 5\sqrt{2}$

Solution

$3\sqrt{2} \times 5\sqrt{2} = 3 \times 5 \times \sqrt{2} \times \sqrt{2}$

$= 15 \times 2$ (because $\sqrt{2} \times \sqrt{2} = 2$)

$= \underline{\underline{30}}$

To simp[illegible] a surd, you need to look f[illegible] qu[illegible]bers that are factors of th[illegible] orig[illegible] [illegible]er.

Ex[illegible]les 2

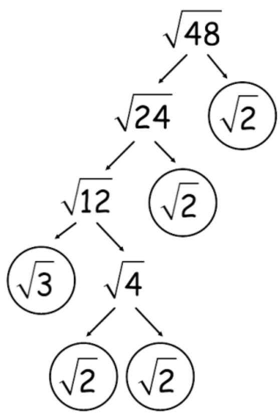
Express $\sqrt{48}$ and [illegible] ir simplest form

Solution

$\sqrt{48} = \sqrt{16 \times 3}$
$= \sqrt{16} \times \sqrt{3}$
$= 4 \times \sqrt{3}$
$= \underline{4\sqrt{3}}$

$\sqrt{98} = \sqrt{2 \times 49}$
$= \sqrt{2} \times \sqrt{49}$
$= \sqrt{2} \times 7$
$= \underline{7\sqrt{2}}$

A[illegible]ative Method (for $\sqrt{48}$): split 48 up into its prime fact[illegible] – see the diagram on the right (note we could split it up [illegible]ver we wanted and would always end up with the same [illegible]swer).



From the diagram, we get:

$\sqrt{48} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times 2 \times \sqrt{3}$
$= 4\sqrt{3}$

Rules of Indices

Basic Rule 1: anything to the power 0 is equal to 1:

e.g. $5^0 = 1, \quad 17^0 = 1, \quad 35627658^0 = 1, \quad x^0 = 1$

Basic Rule 2: anything to the power 1 is equal to itself:

e.g. $5^1 = 5, \quad 17^1 = 17, \quad 35627658^1 = 35627658, \quad x^1 = x$

Key Rule 1: when you multiply two expressions involving powers, you <u>add</u> the numbers in the power: $a^m \times a^n = a^{m+n}$

e.g. $x^3 \times x^4 = \underline{x^7} \qquad y^{-1} \times y^6 = y^{-1+6} = \underline{y^5}$

Key Rule 2: when you divide two expressions involving powers, you take away the numbers in the power: $\frac{a^m}{a^n} = a^{m-n}$

e.g. $a^8 \div a^3 = \underline{a^5} \qquad \frac{m^{10}}{m^8} = \underline{m^2}$

Key Rule 3: when you take one power to the power of another (nested powers), you <u>multiply</u> the numbers in the power: $(a^m)^n = a^{mn}$

e.g. $(x^2)^3 = \underline{x^6} \qquad (a^{-4})^2 = \underline{a^{-8}}$

Example 1

Simplify $\frac{3x^4 \times 8x^8}{6x^2}$

Solution

$$\frac{3x^4 \times 8x^8}{6x^2} = \frac{24x^{12}}{6x^2} \quad \text{(add the powers when multiplying)}$$
$$= \underline{4x^{10}} \quad \text{(take away the powers when dividing)}$$

Example 2

Simplify $(5x^{-3})^2 \times x^8$

Solution

$$(5x^{-3})^2 \times x^8 = 25x^{-6} \times x^8 \quad \text{(multiplying the nested powers)}$$
$$= 25x^2 \quad \text{(adding the powers when multiplying)}$$

Example 3

Simplify $3x^2(x^{-2} + 2x^5)$

Solution

$$3x^2(x^{-2} + 2x^5) = 3x^2 \times x^{-2} + 3x^2 \times 2x^5$$
$$= 3x^{2+(-2)} + 6x^{2+5}$$
$$= 3x^0 + 6x^7$$
$$= \underline{3 + 6x^7}$$

Example 2 – where there is a number before x^2

Factorise $3x^2 + 11x + 6$

Solution

Step 1 – make a list of all possible pairs of numbers – **order matters**.

The two numbers in the bracket will multiply to give **+6**. So possibilities are:

+6 and +1, +1 and [illegible]
+2 and +3 +3 a[illegible]2.

(technically we should also consider –6 and –1 [illegible]nd –[illegible]. As these also multiply to give +6. However in this question we can ignore ne[illegible]bers a[illegible]ere are no negative signs)

Step 2 – To make $3x^2$, we need $3x \times x$, so write 3[illegible]at the start of each brac[illegible]

[illegible])(x)

Step 3 – experiment – try[illegible]t pa[illegible]bers in the bracke[illegible]ltiply brackets out to see if y[illegible]can [illegible]$3x^2 +$ [illegible] $+ 6$.

e.g. firs[illegible]ou m[illegible]y $(3x+6)(x+1)$. But this mul[illegible]s out t[illegible]e $3x^2$ [illegible] so th[illegible]NOT the answer.

[illegible]ow [illegible]u m[illegible] try switching the '6' a[illegible]' abo[illegible]t $(3x+1)(x+6)$. [illegible] th[illegible]plies out to give $3x^2 + 19x + 6$ [illegible]is is NOT the answer.

e.g. [illegible] you might try $(3x +$ [illegible] $+ 3)$ [illegible]ultiplies out to give [illegible]$^2 + 11x + 6$, which is want we [illegible] so this is the answer.

[illegible] **Answer:** $(3x+2)(x+3)$

A[illegible]native Method for [illegible]tor[illegible] Trinomials

[illegible]re is an alternative me[illegible]rising trinomials. The **ad[illegible]ges** [illegible]s me[illegible]od over the 'experiment and check' [illegible]d are that it ought to wo[illegible] first attempt, that we don't need to consid[illegible]ect of changing the order of th[illegible]bers, [illegible] that it will work for *any* trinomial.

The **disadvantages** [illegible] the algebra is more co[illegible]cate[illegible]ou need to be good at finding *all* facto[illegible]of larger numbers, requires a good unde[illegible]ding [illegible] the common factor method, and s[illegible]people find this method harder to remem[illegible]

The Al[illegible]tive Method

[illegible]oris[illegible] $ax^2 + bx + c$:

1. [illegible] all the factors of ac (not just c)
[illegible]ok for the pair of factors that add to make b
[illegible] Split bx up using these factors
4. Factorise each half of the expression using a common factor (single bracket) – you will end up with the same bracket twice.
5. Rewrite as two brackets

It is easiest to understand this method by seeing an example.

Example 1 – (same as Example 2 in last section)

Factorise $3x^2 + 11x + 6$

Solution

Step 1 – list all the factors of 18 (we use 18 because it is 3 × 6)

Order does not matter but positive/negative does.

$+18$ and $+1$, $+9$ and $+2$, [illegible]
$+3$ and $+6$ -3 and -6, [illegible]
-18 and -1, -9 and $-$[illegible]

Step two – look for the pair of factors that add [illegible]ke [illegible]
We need a pair of factors that add to mak[illegible]
This means we use +9 [illegible]2

Step three – split bx up usi[illegible]se fa[illegible]

What this m[illegible]s w[illegible]lit $11x$ up to be $9x + 2x$ (or $2x +$ [illegible]ill work either way)

[illegible]$x^2 + 11x + 6 = 3x^2 + 9x + 2$[illegible]

Step fou[illegible] – fact[illegible]e [illegible] half of the expression by taking ou[illegible] co[illegible]mon factor:

$3x^2 + 11x + 6$ [illegible] $3x^2$ [illegible] $2x + 6$
[illegible]$(x +$[illegible]$) + 2(x + 3)$

[illegible]e how the two brackets are t[illegible]me $(x + 3)$. This will always happen.
If it does not, you have m[illegible]istak[illegible]

[illegible] five – rewrite as two brackets[illegible]

The second brack[illegible] the [illegible] that is the same $(x + 3)$. [illegible] first brack[illegible]s what is 'le[illegible]' w[illegible]n the $(x + 3)$ has been tak[illegible]t.

[illegible]$+ 3) + 2(x + 3)$
$= (3x + 2)(x + 3)$

Final answer: [illegible]$x + 2)(x + 3)$

[illegible]s always, we should still multiply this bracke[illegible]ck out to double check our answer]

Ex[illegible]e 2 – (where the 'guess and check' method would be much less efficient)

Factorise $6x^2 - 11x - 10$

Solution

Step 1 – list all the factors of –60 (we use –60 because it is 6 × –10).

Order does not matter but positive/negative does.

–60 and +1, +60 and –1, –30 and +2, +30 and –2,
–20 and +3, +20 and –3, –15 and +4, +15 and –4,
–12 and +5, +12 and –5, –10 and +6, +10 and –6.

Adding and Subtracting Algebraic Fractions

You can only add and subtract fractions when the denominators are the same. When they are not the same, we have to change the fractions into another fraction that *is* the same.

A quick method for doing this, and the one used in these notes, is known as the **'kiss and smile'** method because of the shape formed when you draw lines between the terms you are combining:

$$\frac{a}{b} \times \frac{c}{d}$$

$$\frac{a}{b}+\frac{c}{d}=\frac{}{bd}$$

Step One (the "smile") – multiply the two bottom numbers together to get the "new" denominator, which will be the same for each fraction.

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{}$$

Step 2a (the first part of the "kiss")

Multiply diagonally the top left and bottom right terms

$$\frac{a}{b}+\frac{c}{d}=\frac{\quad bc}{}$$

Step 2b (the final part of the "kiss")

Multiply diagonally the top right and bottom left terms

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

Step 3 – Insert the add (or take away) sign between the two terms on the top row to get the final answer.

Example 1

Add, giving your answer as a single fraction in its simplest form:

$\frac{m}{5}+\frac{3}{m}, \quad m \neq 0$

Solution

Step one (smile) $\frac{m}{5}+\frac{3}{m}$

Step two (kiss) $\frac{m}{5} \times \frac{3}{m}$

$$\frac{m}{5}+\frac{3}{m}=\frac{}{5m}$$

$$=\frac{m^2+15}{5m}$$

Final answer: $\frac{m^2+15}{5m}$

Perimeter, Area and Volume

Rounding to Significant Figures

Example 1

446·586 → Rounded to 1 significant figure is: **400**
→ Rounded to 2 significant figures is: **450**
→ Rounded to 3 significant figures is: **447**
→ Rounded to 4 significant figures is: **446·6**

Example 2

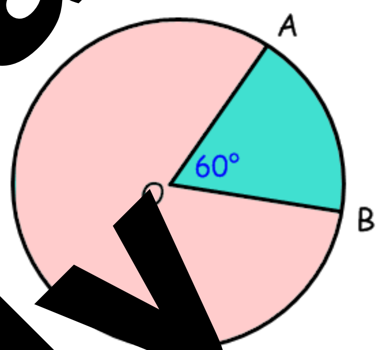
0·00567 → Rounded to 1 significant figure is: **0·006**
→ Rounded to 2 significant figures is: **0·0057**

Arc Length and Sector Area

An **arc** in a circle is a fraction of its circumference. A **sector** of a circle is a fraction of its area.

If you divide a circle into two bits, you get two sectors - a bigger one (major) and a smaller one (minor). In the diagram [illegible]:

- The smaller blue sector OAB is the minor sector, with the minor arc AB.
- The larger pink sector OAB is called the major sector, with the larger major arc AB.



The key step in these questions is to identify the fraction of the circle that is in the question. This depends on the **angle** at the centre of the circle. This fraction is always $\frac{\text{Angle}}{360}$.

Formula. These formulae are **not given** on the National 5 Mathematics exam paper.

Arc length in a circle: Arc length $= \frac{\text{Angle}}{360}\pi d$

Sector area of a circle: Sector Area $= \frac{\text{Angle}}{360}\pi r^2$

You are always allowed to use 3·14 instead of π in calculations.

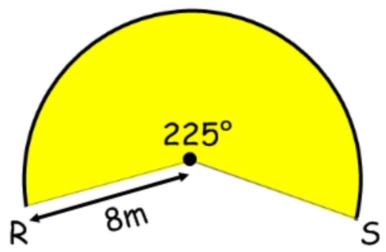
Example [illegible] Arc Length

Find the length of the (major) arc RS in this sector of a circle

Solution

Radius is 8m so diameter is 16m.

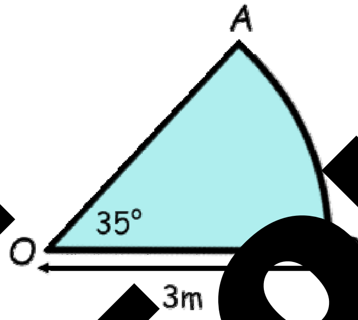
$$\text{Arc length} = \frac{225}{360}\pi d$$
$$= \pi \times 16 \div 360 \times 225$$
$$= 31\cdot41592...$$
$$= \underline{31\cdot4\text{m (1 d.p.)}}$$



Note: units for arc length are just 'normal' units (i.e. not squared or cubic units).

Example 2 – Sector area

Calculate the area of (minor) sector AOB in this diagram.



Solution

$$A = \frac{35}{360}\pi r^2$$
$$= \pi \times 3^2 \div 360 \times 35$$
$$= 2\cdot74889357...$$
$$= \underline{2\cdot75\text{m}^2 \text{ (2 d.p.)}}$$

Note: units for sector area must always be [illegible]ed uni[illegible]

Volumes of Solids

You should know from [illegible]tiona[illegible] to calculate the volume of a [illegible] At N[illegible]nal 5 level, you also need t[illegible] to ca[illegible] the volume of a **pyram**[illegible] T[illegible]out this topic remember that:

- All vo[illegible]ues[illegible]s [illegible] have answered in cubic [illegible]ts (e.[illegible] [illegible]m³, inches³).
- You [illegible] [illegible]ate your unrounded answer be[illegible]undi[illegible] (see page 6).

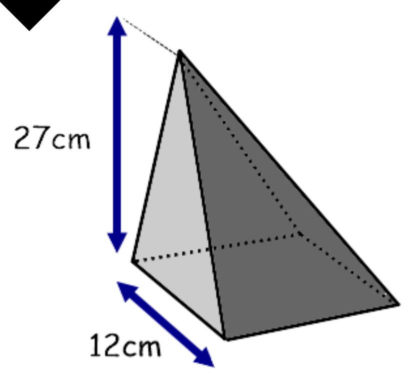
Form[illegible] This for[illegible]is **not** given on the [illegible]al 5 [illegible]s exam paper. [illegible]**f a Prism:** [illegible] Ah V[illegible] = Area of cross section × [illegible]eight

Formul[illegible]his formula is given on th[illegible]at[illegible]na[illegible]athematics exam pape[illegible] **Volume of a Pyramid:** $V = \frac{1}{3}Ah$ Volume $= \frac{1}{3}$ A[illegible]ase × [illegible]ght

---

Example 1 – Pyra[illegible]

The diag[illegible]ow[illegible] pyramid with heigh[illegible] and a squa[illegible]with sides of length 1[illegible]. [illegible]lculate the volume of the pyramid.



Solu[illegible]

[illegible]rea of a square is given by the formula $A = L^2$, so [illegible]e area of the base of this pyramid is 12² = 144cm²

Therefore the volume of the whole pyramid is

$$V = \frac{1}{3}Ah$$
$$= 144 \times 27 \div 3$$
$$= \underline{\underline{1296\text{cm}^3}}$$

Example 1a: using a method involving taking away

Solve algebraically the system of equations $\begin{matrix} 3x - 2y = 11 \\ 2x + 5y = 1 \end{matrix}$

Solution

Step One – Multiply through each equation by the number at the start of the other equation

e.g. in this example, we multiply the top equation by 2, and the bottom equation by 3

$$3x - 2y = 11_{\times 2}$$
$$2x + 5y = 1_{\times 3}$$

$$6x - 4y = 22$$
$$6x + 15y = 3$$

Step Two – Take away the two equations to eli[illegible] terms

$$\cancel{6x} \quad -4[illegible] \quad = [illegible]$$
$$[illegible] \quad +15[illegible] \quad = [illegible]$$
$$[illegible] \quad = 19$$

Step Three – solve th[illegible] equation[illegible]19y = 19, so $y = -1$

Step Four – sub[illegible] for y back into one of the ori[illegible] (either one will do)[illegible] will use the top one:

$$3x [illegible]$$
$$[illegible] 2 \times [illegible]$$
$$[illegible] 11$$
$$3x [illegible] 9$$
$$x = 3$$

[illegible] $x = 3, \quad y = -1$

[illegible] Five – check your answe[illegible] b[illegible] [illegible]stituting into th[illegible]

Al[illegible]native strategy: adding. [illegible] equation has both a positive [illegible] negative [illegible] in [illegible] middle, a better strat[illegible] to get the same coeffici[illegible] the [illegible] to ***add*** the equations.

Example 1b – sam[illegible] Example 1a but using a method [illegible] lving adding

Solve alge[illegible] the system of equati[illegible] $\begin{matrix} [illegible] \\ 2x + [illegible] = 1 \end{matrix}$

Solutio[illegible]

[illegible]ep One – Multiply through each equation by [illegible] number in front of [illegible] the other equation

e.g. in this example, we multiply the top equation by 5, and the bottom equation by 2

$$3x - 2y = 11_{\times 5}$$
$$2x + 5y = 1_{\times 2}$$

$$15x - 10y = 55$$
$$4x + 10y = 2$$

Step Two – Add the two equations to eliminate the y terms

$$15x \quad \cancel{-10y} \quad = 55$$
$$4x \quad \cancel{+10y} \quad = 2$$
$$\underset{15x+4x}{19x} \quad = \underset{55+2}{57}$$

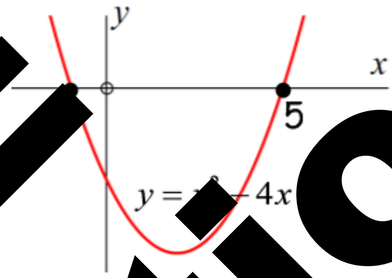
Step Three – solve the resulting equation: $19x = 57$, so $x = 3$

Roots of Quadratic Equations

Definition: the **roots** of a quadratic equation are another word for its solutions.
The roots of a graph of an equation are the points that the graph crosses the x-axis.

Example 1 – from a graph

Using the graph shown, write down the two solutions of the equation $x^2 - 4x - 5 = 0$



Solution

The roots are $x = -1$ and $x = 5$.

Factorising is the simplest way of solving a quadratic equation, but you can only use it when the expression can actually be factorised! See page 2... for help on factorising.

Important – you must rearrange the equation so that is has '= 0' on the right hand side. If you do not do this, you will risk losing all of the marks.

Example 2 - factorising

Use factorising to solve the equation $2x^2 - 6x = 0$

Solution

Step 1 – check that the equation has '= 0' on the right hand side.
On this occasion, it does, so we do not need to do anything more.

Step 2 – factorise the expression

$$2x^2 - 6x = 0$$
$$2x(x - 3) = 0$$

Step 3 – split up into two separate equations and solve

$$2x = 0 \qquad x - 3 = 0$$
$$x = 0, \qquad x = 3$$

Example 3 - Difference of Two Squares

Use factorising to solve the equation $y^2 - 49 = 0$

Solution

Step 1 – check that the equation has '= 0' on the right hand side.
On this occasion, it does, so we do not need to do anything more.

Step 2 – factorise the expression

$$y^2 - 49 = 0$$
$$(y + 7)(y - 7) = 0$$

Step 3 – split up into two separate equations and solve

$$y + 7 = 0 \qquad y - 7 = 0$$
$$y = -7, \qquad y = 7$$

Example 4 – factorising with a coefficient of x^2

Use factorising to solve the equation $2x^2 + 9x - 5 = 0$

The Quadratic Formula

Formula. This formula is <u>given</u> on the National 5 Mathematics exam paper.

The roots of $ax^2 + bx + c = 0$ are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The quadratic formula can be used to solve *any* quadratic equation. We usually use it when you can't factorise the expression.

Important – you <u>must</u> rearrange the equation so that you have = 0 on the right-hand side. If you do not do this, you will risk losing <u>all</u> of the marks.

In the National 5 exam, a clue to use the formula (rather than factorising) is where the question tells you to "**give your answers correct to [illegible] 1) decimal places**" etc. Remember you should always state your unrounded answer before rounding (see page [illegible]).

Example 1

Solve the equation $3x^2 + 2x - 6 = 0$, giving your answers correct to two decimal places.

Solution

Step 1 – check the equation has '= 0' on the RHS. It does so we can proceed.

Step 2 – write down what *a*, *b* and *c* are: $a = 3$, $b = 2$, $c = -6$

Step 3 – substitute into the formula and solve, **being very careful when dealing with negative signs**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-6)}}{2 \times 3}$$

$$x = \frac{-2 \pm \sqrt{4 - (-72)}}{6}$$

$$x = \frac{-2 \pm \sqrt{76}}{6}$$

Essential to remember brackets when putting into the calculator!

$$x = \frac{(-2 + \sqrt{76})}{6} = 1 \cdot 119... = 1 \cdot 12 \text{ (2 d.p.)}$$

$$x = \frac{(-2 - \sqrt{76})}{6} = -1 \cdot 786... = -1 \cdot 79 \text{ (2 d.p.)}$$

If the number under the square root sign works out to be negative, then you will not be able to complete the formula. This means either that:

- You have made a mistake with negative numbers and need to check your working (realistically this is the most likely thing that would have happened in an exam)
- Or the equation has no solution (happens a lot in real life, but less likely in an exam)

Example 2

Solve the equation $2x^2 - 5x - 1 = 3$, giving your answers correct to 2 d.p.

Solution

Step 1 – check the equation has '= 0' on the right-hand side. It does not, so we have to rearrange:

$$2x^2 - 5x - 1 - 3 = 0$$
$$2x^2 - 5x - 4 = 0$$

Step 2 – write down what a, b and c are: $a = 2$, $b = -5$, $c = -4$

Step 3 – substitute into the formula and solve – **be very careful when dealing with negative signs**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-4)}}{2 \times 2}$$
$$x = \frac{5 \pm \sqrt{25 - (-32)}}{4}$$
$$x = \frac{5 \pm \sqrt{57}}{4}$$

Essential to remember brackets when putting into the calculator!

$$x = \frac{(5 + \sqrt{57})}{4} \qquad x = \frac{(5 - \sqrt{57})}{4}$$
$$= 3 \cdot 137... \qquad = -0 \cdot 637...$$
$$= 3 \cdot 14 \text{ (2 d.p.)} \qquad = -0 \cdot 64 \text{ (2 d.p.)}$$

Answer: $x = 3 \cdot 14$ and $x = -0 \cdot 64$

The Discriminant

To solve any quadratic equation, we have to use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since this formula contains $\sqrt{b^2 - 4ac}$ and we can only take the square root of a positive number, the value of $b^2 - 4ac$ is important, and has a special name, the **discriminant**. The symbol Δ can be used for the discriminant.

Formula. This formula is **not** given on the National 5 Mathematics exam paper, though the quadratic formula that contains it is given.
The **discriminant** of a quadratic equation $ax^2 + bx + c = 0$ is given by $\Delta = b^2 - 4ac$

The **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is the number that has to be square rooted to complete the quadratic formula.

A quadratic equation may have two roots (known as **real and distinct** roots), one root (known as a **real and equal** root) or **no real roots**.
Simply put, the discriminant tells us how many real roots there are to a quadratic equation

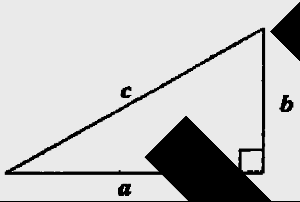
- If the discriminant is positive ($\Delta > 0$), the equation has two **distinct real** roots (also known as **unequal real** roots).

Lengths and Angles

Pythagoras' Theorem

At National 4, you learnt how to use Pythagoras' Theorem to find the length of the third side in a right-angled triangle without needing to measure it.

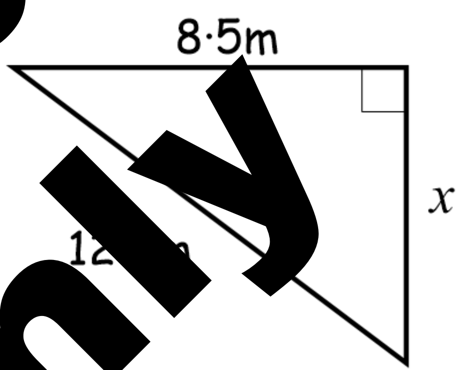
Theorem of Pythagoras. This formula is **not** given o[illegible] National 5 Mathematics exam paper.



There are three steps to any Py[illegible]estion[illegible]
Step One – square t[illegible]gth[illegible]he two sides
Step Two – either add[illegible]away
- T[illegible] the le[illegible]f the longest side (the hyp[illegible]dd the squared n[illegible]
- To [illegible]ngth of a shorter side, **take away** t[illegible] squ[illegible]red numbers.

Step [illegible] ro[illegible]emembering the units i[illegible]r an[illegible]

Example 1 – remi[illegible] of th[illegible] basic method fro[illegible] Nati[illegible]
[illegible]**lculate *x*,** [illegible]**ect to 1 decimal pl**[illegible]



Solut[illegible]
[illegible] finding the length of *x*. [illegible]ller s[illegible]e, [illegible] **take away**.

$x^2 = 12\cdot3^2 - 8\cdot5$[illegible]

$x^2 = 79\cdot04$

$x = \sqrt{79\cdot04}$

[illegible]

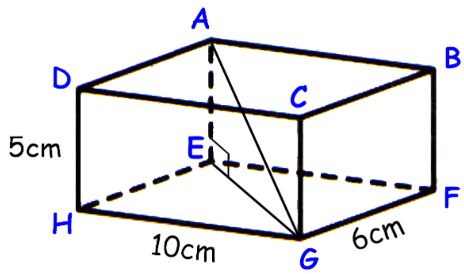
[illegible]d.p.)

At Natio[illegible]l 5 you need to be able to extend your a[illegible]y to u[illegible] Pythagoras in more challen[illegible]g situations:
- In **three-dimensional** diagrams.
- In diagrams involving **circles.**
- [illegible]sing the **converse** of Pythagoras.

[illegible]mple 2 – Pythagoras in 3d

The diagram shows a cuboid with sides of length 5cm, 10cm and 6cm.

Calculate the length of the diagonal AG.



Solution

To find AG, we have to use Pythagoras in triangle AEG. However before we can do this, we must calculate length EG.
First we use Pythagoras in triangle EGH, in which EG is the hypotenuse, and the other sides are 10cm and 6cm.

$$EG^2 = 10^2 + 6^2$$
$$= 136$$
$$EG = \sqrt{136}$$

We could write this answer as a de[illegible] bu[illegible] actually easier for the next step to leave it as a surd.

Now we use Pythagoras in triangl[illegible] AEG, which [illegible]otenuse AG, and other s[illegible] of length 5cm, and $\sqrt{136}$ cm.

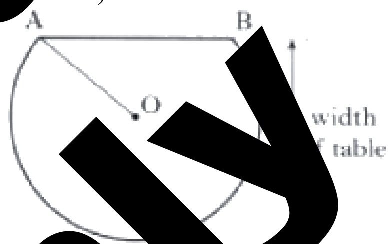
$$AG^2 = \left(\sqrt{136}\right)^2 [illegible]$$
$$= 161$$
$$AG [illegible]$$
$$[illegible] \text{ (1 d.p.)}$$

Definition: a c[illegible] is a straight line going f[illegible]m o[illegible] of the circle to another.

Exa[illegible] Pythago[illegible] circles (past Interm[illegible] 2 ex[illegible]stion)

A t[illegible] is in the shape of part of a c[illegible]
[illegible] of the circle is O.
[illegible] a chord of the circle.
[illegible] 70cm. The radius OA [illegible] 40[illegible]

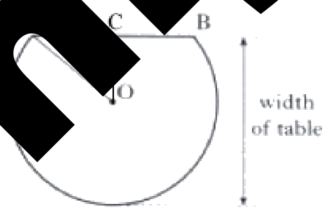
Find the width of the ta[illegible]



[illegible]ution

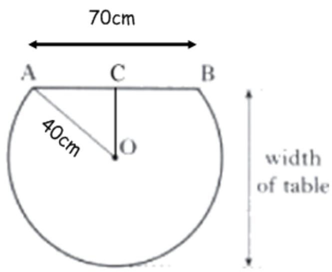
Step one – dra[illegible]ew li[illegible] the diagram to make the diagra[illegible]rical.

In [illegible] on the right, this is [illegible]
This [illegible] that triangle OAC is r[illegible]-angled.



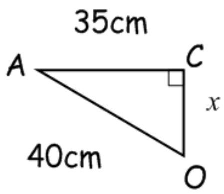
Ste[illegible] two – write in the length of the radius and any other lengths we are told.

OA is 40cm and AB is 70cm.



Step three – identify the lengths in the right-angled triangle, using the fact that the 'new' line splits the chord in half.

In this diagram, AB is 70cm, so AC must be 35cm.



Example 1 (Positive values of sin, cos and tan)

Solve the equation $5\sin x - 2 = 1$ for $0 \le x < 360°$:

Solution:

Step One – rearrange the equation

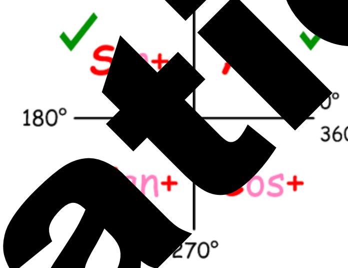
$$5\sin x - 2 = 1$$
$$5\sin x = 1 + 2$$
$$5\sin x = 3$$
$$\sin x = \frac{3}{5}$$

Step Two – find the first solution using sin[illegible]

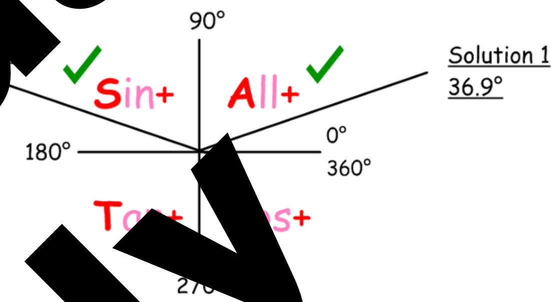
$$x = \sin^{-1}(3 \div 5)$$
$$x = 36 \cdot 9°$$

Step Three – find the second sol[illegible] using CAST

This question inv[illegible]**sin**. [illegible] on the right is [illegible] **posi**[illegible]his means we ti[illegible]AL[illegible]nd SIN quadrant[illegible]



[illegible] bo[illegible]nswers into the [illegible] diag[illegible]hows using symme[illegible]t [illegible]lutio[illegible] 2 is given by 180 – 36[illegible] [illegible]°.



[illegible]wer: $x = 36 \cdot 9°$, $x = 143 \cdot 1$[illegible]

[illegible]le 2 (Negative values of s[illegible]os a[illegible]

Solve the equation 3 cos[illegible]**3 = 1 for $0 \le x < 360°$:**

Solution:

Step One – [illegible] the equation

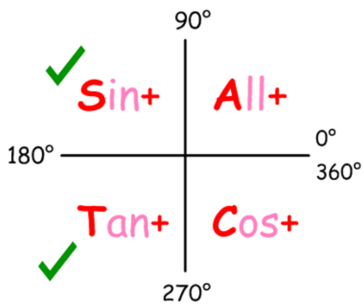
$$3\cos x + 3 = 1$$
$$3\cos x = 1 - 3$$
$$3\cos x = -2$$
$$\cos x = \frac{-2}{3}$$

[illegible]ep Two – find the first solution using cos[illegible]

$$x = \cos^{-1}(-2 \div 3)$$
$$x = 131 \cdot 8°$$

Step Three – find the second solution using CAST

This question involves **cos**. The number on the right is $\frac{-2}{3}$, which is **negative**. This means that cos is NOT positive, so we do NOT tick ALL and COS, instead we tick the SIN and TAN quadrants.



Applications Unit

Trigonometry

Area of a triangle

To find the area of any triangle you need the length of tw[illegible]des and the size of the angle between them.

Formula. This formula is given on the National 5 M[illegible]tics e[illegible] paper.
Area of a Triangle: [illegible] $\sin C$

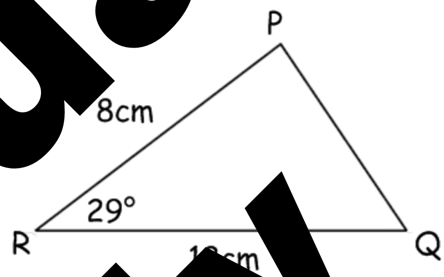
Important: in the formula, a and b mea[illegible]ngths, a[illegible] C means the angle. It is [illegible]ssib[illegible] that you may be given a diagram w[illegible] diff[illegible] ett[illegible]re used. You hav[illegible] to igno[illegible]se letters and relabel the two sides [illegible]rder [illegible] matter) and the an[illegible]etwe[illegible]m as C.

Example

Find the are[illegible] triang[illegible] ound your answer to [illegible]gn[illegible]figures.

Solution

a and [illegible]s, s[illegible] we use $a = 8$, $b = 12$
C is the [illegible] be[illegible] the lengths, so $C = 29°$



[illegible] $\sin C$
$A = $ [illegible] $\sin 29° \div 2$
[illegible] $23 \cdot 27086177...$
[illegible] $23 \cdot 3\text{cm}^2$ (3 s.f.)

[illegible] Rule

Formula. This form[illegible]iven on the National 5 Mathem[illegible]exam[illegible]r.
S[illegible] $\frac{a}{\ldots A} = \frac{b}{\ldots B} = \frac{c}{\sin C}$

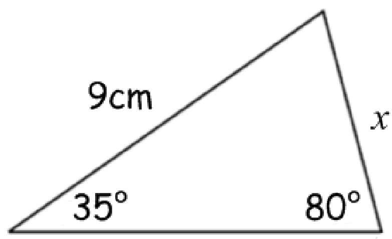
Where [illegible] and c are the lengths of the sides of the [illegible]ngle [illegible]d A, B and C are the angles in the tri[illegible]e. Side a is opposite angle A etc.

Import[illegible] [illegible] answer a question you do not use the formula as it is written. You only need the[illegible] two 'bits': $\frac{a}{\sin A} = \frac{b}{\sin B}$

[illegible]xample 1 – sine rule for lengths

Find the length x in this triangle.

Solution



x cm is opposite 35°, so use $a = x$ and $A = 35°$
9cm is opposite 80°, so use $b = 9$ and $B = 80°$

Vectors

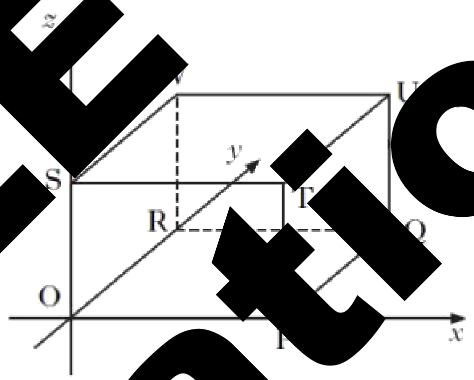
3d Coordinates

We can extend the traditional 2 dimensional Cartesian diagram into 3 dimensions by adding a third axis called the *z* axis which is at right angles to both the *x* axis and *y* axis.

Example (diagram adapted from 2010 Higher exam pa[illegible]

In the diagram on the right, the point U has coordinates (4, 2, 3).

State the coordinates of P, V and Q.



Solution

P is the point (4, 0, 0).
V is the point (0, 2, [illegible]
Q is the point (4, 2, [illegible]

Definition o[illegible] Ve[illegible]r

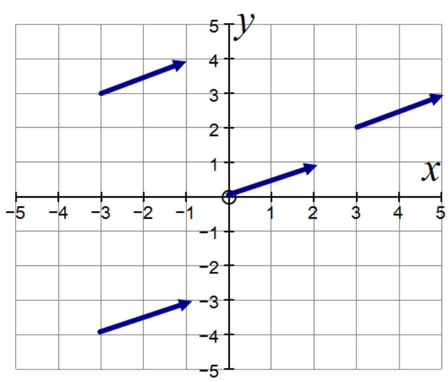
A vector is a qu[illegible] th[illegible] both size and direction. [illegible] can [illegible]esented as an arrow, where the length o[illegible] arrow represents the [illegible]or's s[illegible] now[illegible] a **directed line seg[illegible]** [illegible] the di[illegible]n the arrow is point[illegible] repr[illegible] direction.

[illegible]here are two ways of naming [illegible]

- One way is to re[illegible] vector by a single letter. For [illegible]ce in [illegible]hree examples above, the thre[illegible]ect[illegible] are called **a**, **u** and **x**. In print, w[illegible] bold type letter to represent a ve[illegible]g. [illegible] When handwriting, we [illegible]der[illegible]in place of bold, e.g. a.
- Another way is [illegible]ent a vector using the [illegible]t a[illegible] points. For instance the first vect[illegible] above goes from A to B, and so it coul[illegible] repre[illegible]ted as $\overrightarrow{AB}$. The middle vector [illegible] be represented $\overrightarrow{PQ}$, and the final one wou[illegible] epresented $\overrightarrow{HG}$ (not $\overrightarrow{GH}$).

[illegible]pon[illegible] of a vector

A[illegible]r is described in terms of its **components**, which desc[illegible]e how far the vector moves in the *x* and *y* directions [illegible]tively. For a three-dimensional vector there would be [illegible]ee components, with the third component referring to the *z* direction.



With vectors, the important thing is how the vector moves, not where it begins or starts. All the vectors in the diagram on the right represent the same vector **a**, as both move 2 units in the *x* direction and 1 unit in the *y* direction:

The components of a vector are written in a column. A 2-d vector would be written $\begin{pmatrix} x \\ y \end{pmatrix}$. A 3-d vector would be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. For example the vector $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ is a 3-d vector moving 1 unit in the x direction, 2 units in the y direction and –3 units in the z direction.

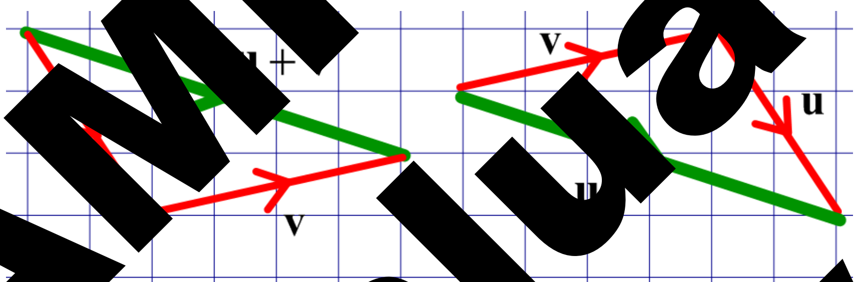
Adding Vectors

We can add vectors to create a **resultant vector**. We can do this in two ways:

- **numerically** by adding their components.

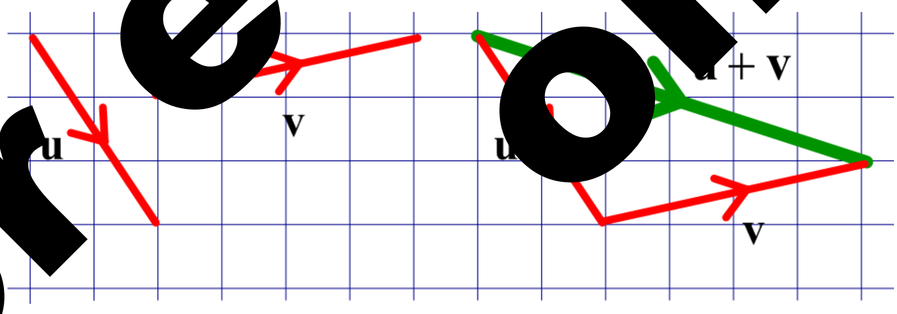
If we have two vectors, $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the resultant vector $\mathbf{a} + \mathbf{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$.

- **In a diagram** by joining them 'nose to tail'.



$\mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $\mathbf{v} + \mathbf{u} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

It does not matter which order you add vectors in: $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$.



$\mathbf{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

In real life, resultant vectors can be used to work out what the combined effect of more than one force pulling on an object will be.

<u>Example 2</u> – find the percentage first

A house cost £240 000 when first bought. One year later its value has appreciated to £250 800.

a) Find the rate of appreciation (in an exam, this part would have 1 mark)

b) If the house continues to appreciate at this rate, what will its value be after a further 4 years?

Solution

a) The increase is 250 800 – 240 000=£10 800

Using the formula, the percentage incre[illegible] gi[illegible]y:

$\frac{10800}{240000}\times 100 = \underline{\underline{4\cdot 5\%}}$

b) Using the quicker method:

Appreciation means i[illegible]e, so [illegible] adding.
100% + 4·5% = 1[illegible]%, [illegible]ve use [illegible]5 in the quicker meth[illegible]
The question is f[illegible]ur y[illegible] so need to repeat 4 times (a pow[illegible]
(Note the question s[illegible]er four years – so we start with £2[illegible]0 800 n[illegible]000).

$250800\times$ [illegible] 299[illegible]65

A[illegible]fte[illegible] fur[illegible] 4 years, the house will [illegible]orth [illegible]000 (3 s.f.)

<u>Example 3 – no st[illegible]alue</u>

[illegible]intage ca[illegible]preciated in value [illegible]% and [illegible]ppreciated in value by 12%. [illegible]h had its value changed over[illegible]

Soluti[illegible]

[illegible]es not matter that we do n[illegible] ma[illegible] value. We can just do t[illegible] mu[illegible]ication c[illegible]ulation with the multipl[illegible]ier[illegible] al[illegible]e.

The multiplier for a 5% de[illegible]ciation is 0·95 (since 100 – 5 = 95)
The multiplier for [illegible]ciation is 1·12 (since 100 [illegible] = 11[illegible]

$0\cdot 95\times 1\cdot 12$ [illegible]

1·064 is th[illegible]tipl[illegible] for a 6·4% appreciati[illegible]

A[illegible]nswer: a 5% depreciation, followed by a[illegible]% de[illegible]iation is equivalent to a <u>6·4%</u> <u>[illegible]ppreciation</u> overall.

> **Tip[illegible]**<u>ays</u> use multipliers and powers in any National 5 percentages exam question.

Co[illegible]ound Interest

[illegible]ompound interest is always an example of **appreciation** (because the amount in the account is always going <u>up</u>), so you <u>always add</u> the amount on each time.

There are two types of questions you may be asked about compound interest:

- If the question asks **how much money** is in the account (the **balance**) you do the appreciation calculation as normal. See Example 1.

Statistics

Scatter Graphs and Line of Best Fit

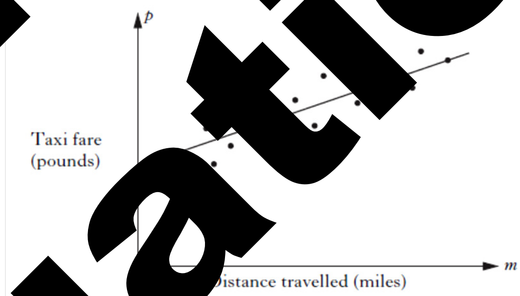
The line of best fit on a scattergraph is a straight line. This means that you can find the equation of a line of best fit using the method ($y = mx + c$) on page 39.

Once you have the equation, you can use the equation t[illegible]mate the value of y when you are told x (or vice versa). At National 5, **you have to us[illegible] e[illegible]on** to get any marks (the question will say this). You *cannot* do it by "looki[illegible]ssing". Any answer without working will get zero marks, even if it happens to be [illegible]

Example 1 (2010 Intermediate 2 Exam Q[illegible]tion)

A scattergraph shows the taxi [illegible] pounds [illegible] plotted against the distan[illegible]avel[illegible] miles. A line of best fi[illegible] dra[illegible]

The equation of the [illegible]t fit is $p = 2 + 1 \cdot 5m$ [illegible] this [illegible]ion to predict the taxi fare [illegible]rney [illegible]iles.



Solution

The j[illegible], so $m = 6$. Using the equatio[illegible]

$p = [illegible] + 1 \cdot [illegible]$

$p = [illegible] 5 \times 6$

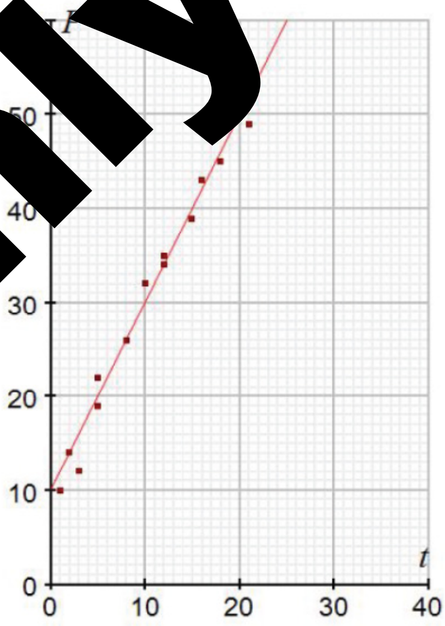
[illegible]11 miles

Examp[illegible]

[illegible] scattergraph shows the [illegible]w[illegible] of [illegible]ndustrial b[illegible]ttery (P) after t hours [illegible] char[illegible]

(a) Find the equation o[illegible] line [illegible]est fit.

(b) Use your equa[illegible] to [illegible]mate the power of a battery that has [illegible]ged for 60 hours.



Solution

(a) We u[illegible]su[illegible]ethod for $y = mx + c$ ([illegible]age 41). O[illegible]asion, since the letters [illegible]nd [illegible] being used, we will use $P = mt + c$.

The y intercept is 10, so $c = 10$.

[illegible]ow choose two points on the line (not necessarily on the original scatter graph) to calculate the gradient.

Two points on the line are (0, 10) and (10, 30). Therefore the gradient is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 10}{10 - 0} = \frac{20}{10} = 2$$

Formula. These formulae are given on the National 5 Mathematics exam paper.

$$\text{standard deviation} = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} \quad \text{or} \quad \sqrt{\frac{\sum x^2 - \left(\sum x\right)^2 / n}{n-1}}$$

Where n is how many numbers are in the list, $\bar{x}$ is the mean and Σ means "add together"

You only need to use one of these formulae. In gener[illegible] is more helpful to just know the method rather than memorising the formula.

Example

a) **Find the mean of these five numbers: [illegible], 3, 9, 6, 5**

b) **Find the standard deviation [illegible] f the same five [illegible] bers**

Solution

a) $\frac{2+3+9+6+5}{5} = 2$[illegible] the m[illegible] s 5

b) You have [illegible] ice o[illegible] methods:

[illegible] hod [illegible] – us[illegible] he formula $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-[illegible]}}$

St[illegible] – Draw up a table sho[illegible] x, x[illegible] nd $(x-x$[illegible]

x	$x-\bar{x}$	$(x-\bar{x})^2$
2		
3		
9		
6		
[illegible]		

Step 2 – C[illegible] te [illegible] table, remembering th[illegible] $\bar{x} =$ [illegible] $= 5$.

St[illegible] – fi[illegible] he total of the final co[illegible]

So $\sum(x-\bar{x})^2 = 30$

[illegible]	[illegible] $\bar{x}$	$(x-\bar{x})^2$
[illegible]	–3	9
3	–2	4
[illegible]9	4	16
6	1	1
5	0	0
TOTAL		**30**

Step 4 – use the formula, remembering that $n = 5$ as there were five numbers.

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{30}{5-1}}$$

$$= \sqrt{\frac{30}{4}} = 2 \cdot 74 \text{ (2 d.p.)}$$

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