National 5
Applications of Mathematics
Revision Notes

Last updated January 2019

Use this booklet to practise working independently like you will have to in an exam.

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back pages) to look it up.

As you get closer to the exam, you should be aiming to use this booklet less and less.

This booklet is for:

- Students doing the National 5 Applications of Mathematics course.
- Students studying one or more of the National 5 Applications of Mathematics units: **Numeracy**, **Geometry and Measures** or **Managing Finance and Statistics**.

This booklet contains:

- The most important facts you need to memorise for National 5 Applications of Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:

- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for the exam.

The key to revising for a maths exam is to do questions, not to read notes. **As well as using this booklet, you should also:**

- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or other exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a Smartphone is on the last page).

Contents

Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

Formulae that <u>are given</u> on the formula sheet in the exam (or in unit assessments)

Topic	Formula(e)	Page Reference
Pythagoras' Theorem	$a^2+b^2=c^2$	See page 51
Gradient	$\text{Gradient}=\dfrac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 54
Circumference of a Circle	$C=\pi d$	See page 58
Area of a Circle	$A=\pi r^2$	See page 58
Volume of a prism	$V=Ah$	See page 62
Volume of a cylinder	$V=\pi r^2 h$	See page 62
Volume of a cone	$V=\dfrac{1}{3}\pi r^2 h$	See page 63
Volume of a sphere	$V=\dfrac{4}{3}\pi r^3$	See page 64
Standard deviation	$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$ or $\sqrt{\dfrac{\sum x^2-\frac{(\sum x)^2}{n}}{n-1}}$	See page 92

Formulae that are <u>not</u> given in the exam (or in unit assessments)

Topic	Formula(e)	Page Reference
Percentage increase and decrease	$\dfrac{\text{increase (or decrease)}}{\text{original amount}}\times 100$	See page 15
Area of a rectangle	$A=LB$	See page 21
Area of a square	$A=L^2$	See page 21
Area of a triangle	$A=\dfrac{BH}{2}$	See page 21
Volume of a cuboid	$V=LBH$	See page 22
Speed, Distance, Time	$S=\dfrac{D}{T}$ $T=\dfrac{D}{S}$ $D=ST$	See page 22
Net Pay	Net Pay = Gross Pay – Total Deductions	See page 70
InterQuartile Range (IQR)	$\text{IQR}=\text{upper quartile} - \text{lower quartile}$ (Q_3-Q_1)	See page 91

In the grid method, each number is split up according to its digits (e.g. 58 is split into 50 and 8; 238 is split into 200, 30 and 8) and then a mini 'tables square' is produced. All the answers in the table square are then added together.

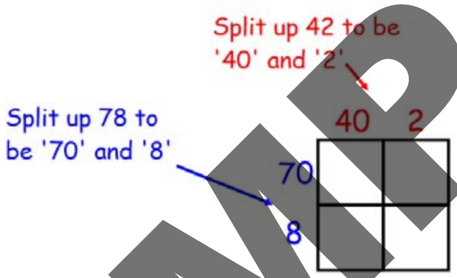
BASIC SKILL EXAMPLE 5: Multiplying two two-digit numbers

Multiply 78 × 42

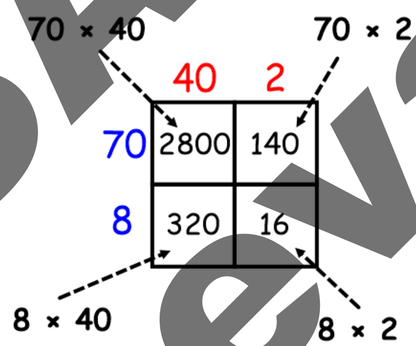
__Solutions__

Method A (Grid Method)

__Step 1:__ construct a multiplication square with two numbers along the side and two numbers along the top.



__Step 2:__ multiply the numbers in each row and column to obtain one number in each of the four smaller squares.



__Step 3:__ add the four numbers to obtain the final answer.

2800 + 140 + 320 + 16 = __3276__

Method B (Long Multiplication)

__Step 1:__ start doing a usual multiplication sum and do 78 × 2 using the usual method.

```
  78
× 42
 1
 156
```

__Step 2:__ the next line will be for 78 × 4. However, because the sum should really be 78 × 40, we write one zero in the units column.

```
  78
× 42
 1
 156
   0
```

__Step 3:__ complete the sum 78 × 4 usual the usual method.

```
  78
× 42
 1
 156
 3
3120
```

__Step 4:__ add the two answers to obtain the final answer.

156 + 3120 = __3276__

These methods can be extended to any multiplication sum, including multiplication of three- (or more)-digit numbers, or multiplication of decimals.

__Assessment Style Example 1: Multiplying a three-digit number and a two-digit number__

It costs £56 to cover one square metre of pathway with concrete. Calculate the cost to cover a path measuring 247m².

BASIC SKILL EXAMPLE 1: Finding the Percentage

Out of 1250 pupils, 475 get to school by bus. What percentage is this?

Solution

As a fraction, this is $\frac{475}{1250}$.

To change this to a percentage divide and then multiply by 100:

475 ÷ 1250 × 100 = 38%

Without a calculator, the calculation can be found using equivalent fractions. Multiply and divide the top and bottom by the same number to obtain the number 100 on the bottom of the fraction. The number on the top is then the percentage.

BASIC SKILL EXAMPLE 2: Finding the Percentage (non-calculator)

Darren baked 20 cakes. 13 of these cakes are carrot cakes. What percentage of cakes are carrot cakes?

Solution

The fraction of carrot cakes is $\frac{13}{20}$. We need to change this into a percentage. To obtain the number 100 on the bottom of the fraction we need to multiply both top and bottom by 5:

$\frac{13^{\times 5}}{20_{\times 5}} = \frac{65}{100}$, so the percentage is 65%.

More difficult questions ask you to find the percentage increase or decrease. In these questions, you always have to work out the percentage of the **original** amount.

Formula: not given on the formula sheet in National 5 assessments

$$\text{Percentage increase/decrease} = \frac{\text{change}}{\text{original amount}} \times 100$$

BASIC SKILL EXAMPLE 3: Finding the Percentage Increase or Decrease

The temperature in an oven was 180°C. It went up to 207°C.
What was the percentage increase in temperature?

Solution

Step one: what is the increase? 207 – 180 = 27°C.

Step two: write as a fraction of the original amount:

Original amount was 180°C, so as a fraction this is $\frac{27}{180}$.

Step three: divide and multiply by 100 to change to a percentage:

27 ÷ 180 × 100 = 15%

For National 5 Numeracy assessment questions, it is likely that the numbers for a percentage question will not be stated in the question. Instead there might be a table, graph or scale to read to determine the numbers.

Assessment Style Example 1

The table on the right shows the numbers of student vets at four Scottish Universities.

Calculate the percentage of the vet students who are women.

University	Men	Women
Edinburgh	110	100
Glasgow	214	223
Dundee	120	197
St Andrew's	132	121

Solution

Total number of women at all four Universities = 100 + 223 + 197 + 121 = 641.
Total number of students = 100 + 223 + 197 + 121 + 110 + 214 + 120 + 132 = 1217.

The fraction of women is $\frac{641}{1217}$. We convert this to a percentage as usual:

641 ÷ 1217 × 100 = 52·67… = 52·7% (1 d.p.)

In National 5 exam percentage questions, you could be asked to increase or decrease an amount by a percentage – this will usually be either **compound interest**, or **appreciation** or **depreciation**. , For every question, there is a longer way and a quicker way to do it. Use the one you are happiest with. In the examples below, the quicker method will be preferred.

Percentage	Longer method	Quicker method
3% increase	Multiply by **0·03**, then add answer on	*[100% + 3% = 103%]* Multiply by **1·03**
3% decrease	Multiply by **0·03**, then take answer away	*[100% – 3% = 97%]* Multiply by **0·97**
2·4% increase	Multiply by **0·024**, then add answer on	*[100% + 2·4% = 102·4%]* Multiply by **1·024**
4·5% decrease	Multiply by **0·045**, then take answer away	*[100% – 4·5% = 95·5%]* Multiply by **0·955**

Definition: Appreciation is an increase in value; **Depreciation** is a decrease in value.

When appreciation or depreciation is repeated, using powers is a quicker method.

BASIC SKILL EXAMPLE 4: Appreciation and Depreciation

Peterburgh has a population of 30 000. Its population depreciates by 15% per year. Calculate its population after two years.

Solution

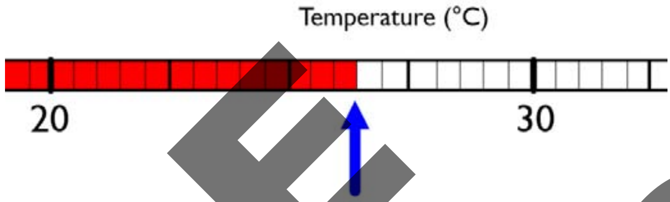
Depreciation means decrease, so we will be taking away.
100% – 15% = 85%, so we use **0·85** in the quicker method.
The question is for two years so need to repeat 2 times (a power of 2).

Longer method	Quicker method
Year 1: 0·15×30000 = 4500 30000 – 4500 = 25500	
Year 2: 0·15×25500 = 3825 25500 – 3825 = 21675	$30000 \times 0{\cdot}85^2 = 21675$
Answer: 21 675	**Answer:** 21 675

The next example goes into this question in a lot of detail. Many people do not need this level of detail and can just 'do it'. However, the question is explained in detail to offer a method to anybody who struggles with this type of question.

BASIC SKILL EXAMPLE: Reading a Scale

The temperature in a medical store room is measured using a thermometer.



The diagram on the right shows the thermometer.
State the temperature in the store room.

Solution

The two numbers marked are 20°C and 30°C. This is a difference of 10°C.
There are 20 minor division between 20°C and 30°C.

Using the formula above, we are 'going up in':
Difference between marked numbers ÷ Number of minor divisions between marked numbers

$= 10°C \div 20 = 0{\cdot}5°C$

Counting up (13 minor divisions) in 0·5°C from 20°C gives us a measurement of 26·5°C.
Note: The diagram in this question is also used in Assessment Style Example on page 35.

Ratio

As well as fractions and percentages, another way to describe the proportions in which quantities are split up is with **ratio**. Ratios consist of numbers separated by a colon symbol e.g., 2:3, 4:1, 3:2:4.

For example, it might be said that a shade of purple paint is made by mixing red paint and blue paint in the ratio 4:5. This means that for every 4 litres (or spoonfuls, tins, gallons…) of red paint, you must add 5 litres (or spoonfuls, tins, gallons…) of blue paint to get the correct shade of purple.

BASIC SKILL EXAMPLE 1: sharing in a ratio

Dave and Jim share £7200 in the ratio 7:2. Calculate how much money they each get.

Solution

Dave gets 7 shares of the money, Jim gets 2 shares.
This means that there are **9 shares** in total.

Dave gets 7 out of 9 shares:

$\frac{7}{9}$ of £7200

$= 7200 \div 9 \times 7$

$= £5600$

Jim gets 2 out of 9 shares:

$\frac{2}{9}$ of £7200

$= 7200 \div 9 \times 2$

$= £1600$

Dave gets £5600 and Jim gets £1600.

Geometry and Measures Unit

Measurement

Converting Measurements, including Time

You will be expected to convert between units of measurement. To do this you should be familiar with all the key facts in the table below. Many of these should be basic to you.

Length
1 kilometre (km) = **1000** metres (m)
1 metre (m) = **1000** millimetres (mm)
1 metre (m) = **100** centimetres (cm)
1 centimetre (cm) = **10** millimetres (mm)

Volume
1 litre (l) = **1000** millilitres (ml)
1 litre (l) = **1000** cubic centimetres (cm^3)
1 millilitre (ml) = **1** cubic centimetre (cm^3)

Weight
1 tonne (t) = **1000** kilograms (kg)
1 kilogram (kg) = **1000** grams (g)
1 gram (g) = **1000** milligrams (mg)

Time
1 normal year = **365** days
1 leap year = **366** days
1 year = **12** months
1 hour = **60** minutes
1 minute = **60** seconds

BASIC SKILL EXAMPLE 1: Converting measurements (tonnes and kilograms)

Convert 125700kg into tonnes.

Solution
1 tonne = 1000kg, so we divide by 1000. 125700 ÷ 1000 = 125·7 tonnes

BASIC SKILL EXAMPLE 2: Converting measurements (hours and minutes)

Convert 8·35 hours into hours and minutes.

Solution
8·35 hours = 8 hours + 0·35 hours = 8 hours ___ minutes.
We need to change 0·35 hours into minutes.

1 hour = 60 minutes, so we multiply by 60 to change hours into minutes.

0·35 × 60 = 21 minutes.
So 8·35 hours = 8 hours 21 minutes

BASIC SKILL EXAMPLE 3: Converting measurements (hours and minutes)

Convert 3 hours 47 minutes into hours.

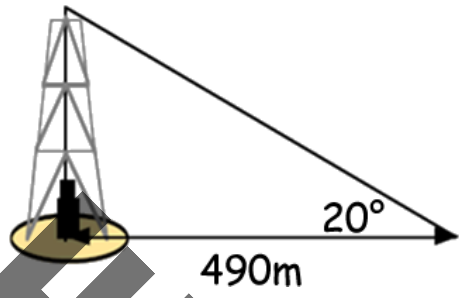
Solution
We need to change 47 minutes into hours.

1 hour = 60 minutes, so we divide by 60 to change minutes into hours.
47 ÷ 60 = 0·78333… (keep at least 3 decimal places, preferably more)
47 minutes = 0·783 hours, so 3 hours 47 minutes = 3·783 hours (rounded to 3 d.p.)

Assessment Style Example

The diagram on the right is a sketch of an oil rig in a desert.

Using a suitable scale, make a scale drawing of the triangle in the diagram and use it to find the real-life height of the oil rig.



Solution

Step One: choose the scale

We need to choose a scale in the form 1cm = _______ m.

If we use the scale 1cm = 10m, the longest edge would be 49cm long which is too long to fit on the page.

We could choose to use a scale of **1cm = 20m**, so the scale factor is 20. However, this is a random choice – other scales are possible.

Step Two: calculate the lengths needed for the drawing (the angles stay the same).

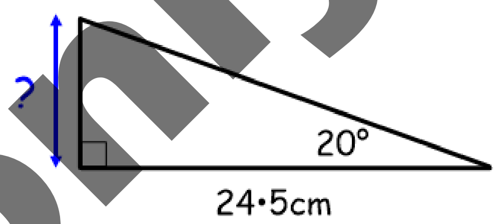
> **Fact:** To find out how long a line should be on the page, **divide** the real-life length by the scale factor.

There is only one real-life length (490m).

490 ÷ 20 = 24·5cm.

Step Three: draw your diagram accurately.

We draw a triangle on our page with a horizontal length of 24·5cm (with an allowed tolerance of ±1mm), and angles of 90° and 20° (with an allowed tolerance of ±1°). This is shown in the diagram on the right, however the actual diagram printed on this page will not be the exact size.



Step Four: use your diagram to calculate the real-life lengths.

> **Fact:** To find out a real-life length, **multiply** the length on the page by the scale factor.

We now measure the height of the oil rig on the page (indicated by a question mark in the diagram). If the diagram had been drawn perfectly, you should get a measurement of 8·9cm.

To find a real-life distance, we multiply the length on the page by the scale factor. The real-life distance is then 8·9 × 20 = **178 metres**.

What should an exam question look like?

In assessments for National 5 Applications, container packing questions should always:

- Involve packing smaller items into larger containers. The larger and smaller containers will all be the same size.
- Ask you to find the maximum possible number of smaller items that can fit into each larger container.
- Be set in a real-life context.

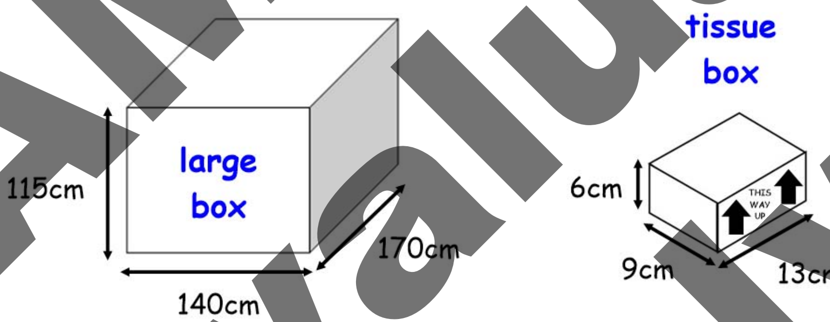
You **might** also have to:

- Identify different ways that the smaller containers could be turned around and determine how this affects the maximum (see **Basic Skill Example 2** on page 42, and Assessment Style Example on page 43).
- Convert between units of measurement (see Assessment Style Example on page 43).

(There are many ways questions may be adapted and so this list can never cover everything).

BASIC SKILL EXAMPLE 2: container packing when smaller items are all the same size

Tissue boxes have dimensions 13cm × 9cm × 6cm.
They are being packed into a larger box with dimensions 115cm × 140cm × 170cm.



All the tissue boxes must be aligned in the same direction. They must be packed upright.
Calculate the maximum number of tissue boxes that will fit in the larger box.

Solution

Step One: identify the different ways that the smaller boxes can be stacked.

The tissue boxes must be packed upright. This means that the two vertical heights (6cm on the tissue box, and 115cm on the larger box) must be lined up.

We can make a table showing the two ways that the tissue boxes can be stacked. The numbers in the top row are the dimensions of the larger box. The numbers in the other rows are the dimensions of the smaller box showing how they line up with the larger one. *The only number that can go in the 115cm column is 6cm, because the numbers 6 and 115 must line up. The other numbers (9cm and 13cm) can go either way around in the other two columns.*

	115cm	**140cm**	**170cm**
Method 1	6cm	9cm	13cm
Method 2	6cm	13cm	9cm

(continued on next page)

(Basic Skill Example 2 continued)
Step Two: work out how many boxes will fit in along each edge.

We now need to do a division sum (longer length ÷ shorter length) for each pair of numbers. Each answer must be rounded **down** (not up) to the nearest whole number. The decimal parts of the answers are shown in brackets and are ignored.

	115cm	140cm	170cm
Method 1	115 ÷ 6 = **19**(·16…)	140 ÷ 9 = **15**(·55…)	170 ÷ 13 = **13**(·07…)
Method 2	115 ÷ 6 = **19**(·16…)	140 ÷ 13 = **10**(·76…)	170 ÷ 9 = **18**(·88…)

Step Three: for each arrangement, multiply the three values to find the total number of tissue boxes that will fit.

Method 1: 19 × 15 × 13 = 3705 tissue boxes.

Method 2: 19 × 10 × 18 = 3420 tissue boxes.

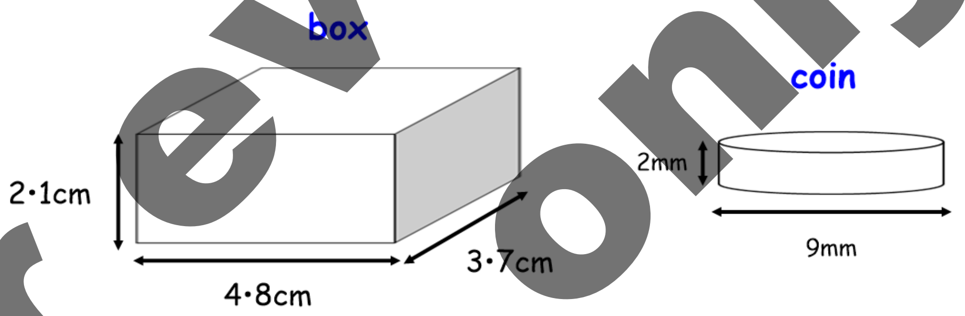
Step Four: conclusion.

The maximum number of tissue boxes that can be fitted into the larger box is 3705 because the other arrangement gives 3420, which is less than 3705.

When the smaller shape is a cylinder, you may only be given two dimensions. However, a cylinder is actually a 3-dimensional shape, so we still need to use three numbers: it just happens that two of the dimensions are the same, as the diameter is both the length and breadth of the cylinder.

Assessment Style Example

Chocolate coins are in the shape of a cylinder with diameter 9mm and height 2mm as shown. They must be packed in piles inside a cuboid box with dimensions 2·1cm × 4·8cm × 3·7cm.



All of the coins must be aligned in the same direction. Find the maximum number that can be packed into one box.

Solution

The diameter of the cylinder is 9mm. The three dimensions of the cylinder are therefore 2mm, 9mm and (again) 9mm.

The dimensions of the cuboid are in centimetres, and the dimensions of the cylinder are in millimetres. We cannot mix units within a question, so we convert one set of units. The dimensions of the cuboid in millimetres are 21mm, 48mm and 37mm.

What should an exam question look like?

In assessments for National 5 Applications of Mathematics, task planning questions should always:

- Involve a precedence table.
- Be set in a real-life context.

You **might** also have to:

- Calculate the minimum time for the task (see **Assessment Style Example** on page 46).

(There are many ways questions may be adapted and so this list can never cover everything).

When given a precedence table, you are likely to be asked to use the information to create an activity network. A pre-drawn diagram may be provided for you.

You can complete these tasks by experimenting, but here are guidelines that may help:

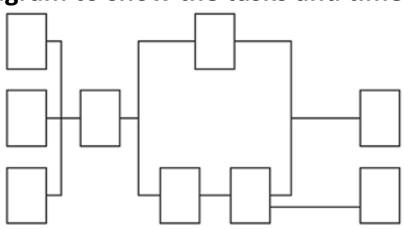
1) Try to fill tasks in on the diagram in order from the start of the task to the end (the 'start' is usually, but not necessarily always, the left-hand side of the diagram).
2) Any task for which preceding task is listed as 'None' will be at the very 'start' of the diagram.
3) Once you have inserted a particular letter (for example 'B') into the diagram, cross that letter off on all occasions it appears in the 'Preceding Task' column. Once all the instances of that particular letter have been crossed off in the 'Preceding Task' column, it means that letter is ready to be added into the diagram.

Assessment Style Example (2014 SQA exam question, slightly adapted)

The Clark family are having a new kitchen fitted by a company called Kitease. Kitease provide a team of workers to install the kitchen. The precedence table shows the list of tasks and the time required for each.

Task	Detail	Preceding Task	Time (hours)
A	Plaster walls	B, C, D	8
B	Begin electrics	None	3
C	Build cupboards	None	5
D	Begin plumbing	None	2
E	Fit wall cupboards	A	6
F	Finish electrics	E, I	4
G	Finish plumbing	I	3
H	Fit floor cupboards	A	5
I	Fit worktops	H	3

(a) Complete the diagram to show the tasks and times in the boxes.

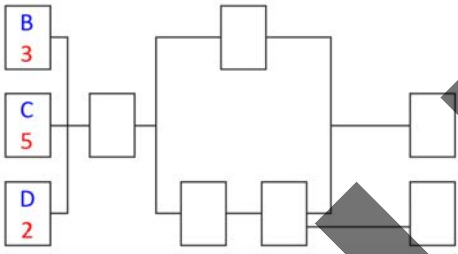


(b) Calculate the minimum time in which this kitchen could be installed.

Solution

(a) Step One: identify the tasks with preceding task 'None' and insert these in the 'first' boxes.

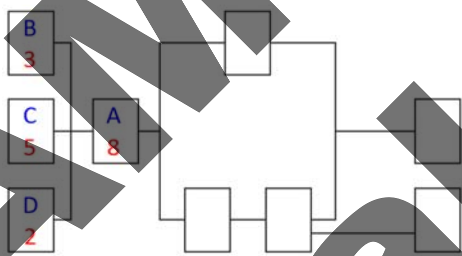
In this question, these are 'B', 'C' and 'D'. Insert them into the diagram and cross them off in the 'preceding task' column in the table:



Task	Detail	Preceding Task	Time (hours)
A	Plaster walls	XXX	8
B	Begin electrics	None	3
C	Build cupboards	None	5
D	Begin plumbing	None	2
E	Fit wall cupboards	A	6
F	Finish electrics	E, I	4
G	Finish plumbing	I	3
H	Fit floor cupboards	A	5
I	Fit worktops	H	3

Step Two: identify the tasks whose preceding tasks are crossed out, and insert these into the table and cross them off in the 'preceding task' column:

In this question, the next task to be inserted is 'A':

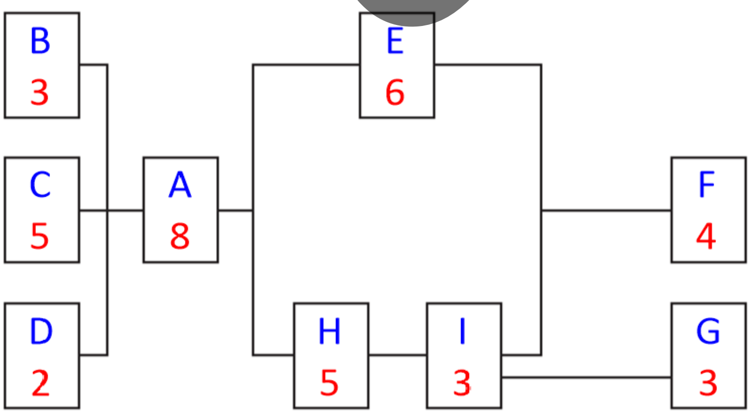


Task	Detail	Preceding Task	Time (hours)
A	Plaster walls	XXX	8
B	Begin electrics	None	3
C	Build cupboards	None	5
D	Begin plumbing	None	2
E	Fit wall cupboards	X	6
F	Finish electrics	E, I	4
G	Finish plumbing	I	3
H	Fit floor cupboards	X	5
I	Fit worktops	H	3

Step Three: repeat step two.

In this question, the next tasks to be inserted are 'E' and 'H'. However you have to use the table to decide which way around they go. The one that goes in the 'bottom' line has another task that comes straight after it. You have to look at the table to work out which task this must be. It turns out to be 'I'.

The fully completed diagram looks like this:



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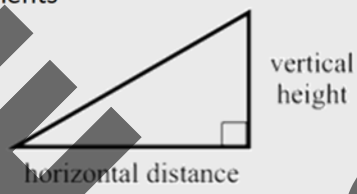
Gradient

The **gradient** of a slope is a measure of its steepness. If the gradient is larger, the slope is steeper.

Formula: given on the formula sheet in National 5 assessments

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$

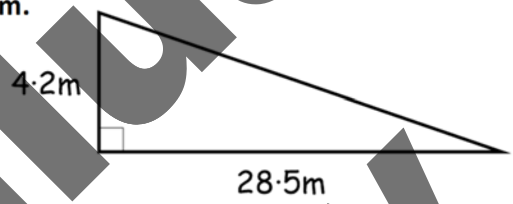


In some situations, it makes sense to talk about positive or negative gradients. A **positive** gradient (e.g. 2, $\frac{1}{2}$, 0·364) means the line slopes **upwards**. A **negative** gradient (e.g. –2, $-\frac{1}{2}$, –0·364) means the line slopes **downwards**. For example:

- A gradient of 2 means 'along 1, up 2'.
- A gradient of –3 means 'along 1, down 3'
- A gradient of $\frac{3}{4}$ means 'along 1, up $\frac{3}{4}$', but it is more expressed as 'along 4, up 3'

BASIC SKILL EXAMPLE: How to calculate the gradient of a slope

Calculate the gradient of the slope in the diagram.



Solution

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$
$$= \frac{4 \cdot 2}{28 \cdot 5}$$
$$= \underline{\underline{0 \cdot 147}} \text{ (3 d.p.)}$$

What should an exam question look like?

In assessments for National 5 Applications of Mathematics, gradient questions should always:

- Be set in a real-life context.
- Require you to calculate a gradient from a diagram.

You might also have to:

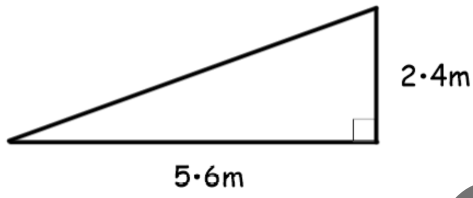
- Make and explain a decision about whether a gradient is within a given limit **without** a calculator (see **Assessment Style Example 1**).
- Make and explain a decision about whether a gradient is within a given limit **with** a calculator (see **Assessment Style Example 2**).
- Calculate one or both of the horizontal or vertical distances before the calculation (e.g. using subtraction or Pythagoras' theorem) (see **Assessment Style Example 2**).
- Work out dimensions when given the gradient (see **Assessment Style Example 3**).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

(Non-calculator) A ramp in on a snowboarding course is shown in the diagram.

To be classed as 'difficult' the ramp should have a gradient higher than $\frac{2}{5}$. Determine whether the ramp is classed as 'difficult'. Justify your answer.



Solution

First use the usual formula to calculate gradient:

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}} = \frac{2 \cdot 4}{5 \cdot 6}$$

Now simplify the fraction. The easiest way to 'remove' the decimals is to multiply the top and bottom of the fraction by 10 first:

$$\text{Gradient} = \frac{2 \cdot 4^{\times 10}}{5 \cdot 6_{\times 10}} = \frac{24^{\div 8}}{56_{\div 8}} = \frac{3}{7}$$

The gradient is $\frac{3}{7}$. To determine whether this is a 'difficult' ramp, we must show whether $\frac{3}{7}$ is higher than $\frac{2}{5}$. We use the method for comparing fractions outlined on page 19, where we change both fractions so that they have the same denominator (number on the bottom).

A common multiple of 7 and 5 is 35, so we change both denominators to 35:

$\frac{3^{\times 5}}{7_{\times 5}} = \frac{15}{35}$, $\frac{2^{\times 7}}{5_{\times 7}} = \frac{14}{35}$. This shows us that $\frac{3}{7}$ is higher.

The *decision* is that Yes, the ramp is classed as 'difficult'.

The following *justifications* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

- Yes, because the gradient is $\frac{15}{35}$ which is higher than $\frac{14}{35}$.
- Yes, because $\frac{15}{35} > \frac{14}{35}$.
- Yes, because it is $\frac{1}{35}$ over the limit.

You would **not** get a mark for this justification:

- Yes, because the gradient is $\frac{3}{7}$ which is higher than $\frac{2}{5}$ *(because it is not possible to compare two fractions that haven't got the same denominator)*

Volumes of 3-d Shapes

Definitions
A **prism** is a 3d solid "with a uniform cross-section". In everyday language, this means that is the same shape all the way along.
The **cross-section** is the shape at either end (and throughout the middle) of a prism.

Formula: <u>given</u> on the formula sheet in National 5 assessments

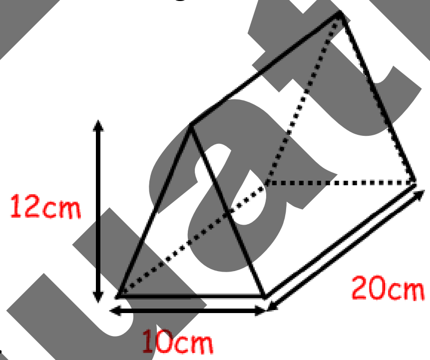
Volume of a prism: $V = \text{Area of cross-section} \times \text{height}$

$V = Ah$

You should also know the important formulae for areas of 2-d shapes given on page 21.

BASIC SKILL EXAMPLE 1: Volume of a (triangular) prism

Calculate the volume of this prism, whose cross section is a triangle.



<u>Solution</u>

The height of this prism is the distance from one (triangular) end to the other.

In this shape, the height is 20cm.

<u>Step 1:</u> Work out the area of the cross-section

In this shape, the cross-section is a triangle.

The formula for the area of a triangle is $A = \frac{BH}{2}$.

Important: you will use a different formula in each question, depending on whether the cross section is a rectangle, square, triangle, circle, semicircle etc.

$$A_{triangle} = \frac{BH}{2}$$
$$= 10 \times 12 \div 2 = 60\text{cm}^2$$

<u>Step 2:</u> Use the formula to find the volume

$$V = Ah$$
$$= 60 \times 20$$
$$= \underline{1200\text{cm}^3}$$

A cylinder is a special example of a prism with a circular cross-section. The method above can be adapted to derive a formula for the volume of a cylinder.

Formulae: <u>given</u> on the formula sheet in National 5 assessments

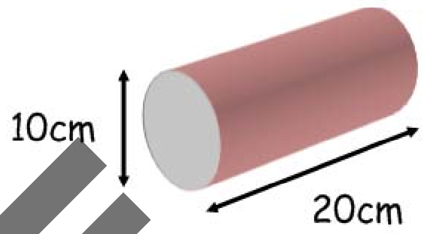
Volume of a Cylinder: $V = \pi r^2 h$

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

BASIC SKILL EXAMPLE 2: Volume of a cylinder

Calculate the volume of this cylinder.



Solution

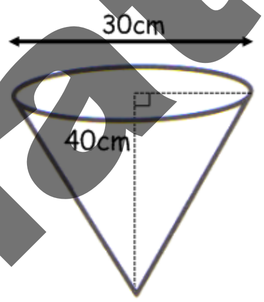
Diameter is 10cm, so radius is 5cm.

$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 20 \quad (\text{ or } \pi \times 5 \times 5 \times 20\ )$$
$$= 1570 \cdot 796327....$$
$$= \underline{1570 \cdot 8\text{cm}^3}\ (1\text{ d.p.})$$

In the cone formula, the 'height' refers to the perpendicular height (the one that goes straight up) and not any sloping heights.

BASIC SKILL EXAMPLE 3: Volume of a cone

Calculate the volume of this cone.



Solution

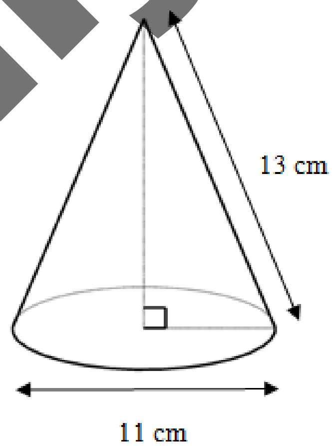
Diameter is 30cm, so radius is 15cm.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \pi \times 15^2 \times 40 \div 3 \quad (\text{ or } 1 \div 3 \times \pi \times 15^2 \times 40\ )$$
$$= 9424 \cdot 777961....$$
$$= \underline{9424 \cdot 8\text{cm}^3}\ (1\text{ d.p.})$$

If a sloping height is given rather than the perpendicular height, Pythagoras must be used to obtain the perpendicular height.

Assessment Style Example 1

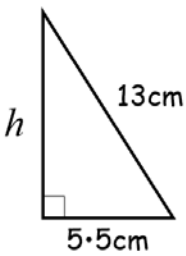
Metal parts for a machine are made in the shape of a cone, with diameter 11 cm and slant height 13 cm, as shown in the diagram.
There are 16 litres of (melted) metal. Calculate how many metal parts can be made.



Solution

The radius of the cone is 5·5cm (half of 11cm).

The radius, slant height and perpendicular height form a right-angled triangle as shown in the diagram below, in which the perpendicular height is labelled *h*.



We find *h* using Pythagoras:

$$h^2 = 13^2 - 5 \cdot 5^2$$
$$= 138 \cdot 75$$
$$h = \sqrt{138 \cdot 75}$$
$$= 11 \cdot 779...$$
$$= 11 \cdot 8\text{cm}\ (1\text{ d.p.})$$

(continued on the next page)

Assessment Style Example 2 – profit or loss from a sale (adapted from 2015 exam question)

Megan buys 300 shares for a total price of $1020.

When she decides to sell them, the price is $3·45 per share and Megan must also pay a fee of 2½% of the selling price.

Determine whether she has made a profit or a loss and calculate how much this is.

Solution

Income (when selling):
Selling price = 300 × 3·45 = $1035.
Tax = 2½% of $1035
= 0·025 × 1035
= 25·875
= $25·88.
Income = 1035 – 25·88
= $1009·12

Expenditure (when buying):
$1020

Income – Expenditure = 1009·12 – 1020 = –$10·88.
The decision is that Megan makes a **loss**, and that the loss is $10·88.

Pay

Definition: deductions are amounts of money that get taken off your pay. The most common deductions are:

- **Income tax**: tax taken off your pay and paid directly to the Government. See page 69 for more details about how tax is calculated.
- **National Insurance**: money taken off your pay to go towards paying for services such as hospitals. This is usually a percentage of pay.
- **Superannuation** (pension): money taken off your pay and kept in a fund to pay you a pension when you eventually retire. This is usually a percentage of pay.

A **bonus** is paid to staff when they reach certain targets within a period of time. One example of a type of bonus is commission.

Definition: commission is a bonus paid to a salesperson based on their sales. It is usually a percentage of their sales.

BASIC SKILL EXAMPLE 1: Commission

Andrew Holmes is a car salesman. He is paid monthly commission of 2·4% on all his sales over £24 000. Calculate his pay in a month where he sells £118 500 worth of cars.

Solution

Sales on which commission is payable = 118 500 – 24 000 = £94 500

Commission = 2·4% of £94 500
= 0·024 × 94 500
= £2268

- Calculate overtime payment (see **Assessment Style Example 1** on page 71).
- Calculate commission payments (see **Assessment Style Example 2** on page 71).
- Calculate the tax and/or pension payable on the income (see **Assessment Style Example 2** on page 71 for pension payments; and the next section for tax).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

Rachel Frost is a radiographer.

- **She works a basic 40-hour week and is paid a basic rate of £14·50 per hour.**
- **Any overtime is paid at time and a half, except when she works on Sundays when she gets paid Double Time instead.**

One week Rachel worked a total of 48 hours, plus an additional 5 hours on Sunday.
Her total deductions are £189·80.
Calculate Rachel's net pay for that week.

Solution

Rachel works a 48-hour week and her basic hours are 40 hours, so she worked 8 hours overtime.

Basic pay: £14·50 × 40 = £580
Overtime pay: £14·50 × 8 × 1·5 = £174
Sunday pay (double time): £14·50 × 5 × 2 = £145

Gross pay: £580 + £174 + £145 = £899

Net pay: £899 – £189·80 = £709·20

Assessment Style Example 2

Harvey Robertson works in sales. Part of his wage-slip is shown.

- **He gets paid 4·5% commission on all items he sells over £850.**
- **He pays 3% of his gross salary into his pension.**

Calculate Harvey's net pay in a month when he sells £3200 worth of items.

Name	Employee number	Week	Date	Tax Code	NI Number
Harvey Robertson	4303	26	26/06/2012	570H	BW 91 44 65 C

Basic Pay	Overtime	Bonus	Commission		Gross Pay
£325.40	£26.00	£50.00			
National Insurance	Income Tax	Pension	Other		Total Deductions
£45.30	£75.20		£0.00		
					Net Pay

Solution

Sales on which commission is payable = 3200 – 850 = £2350.
Commission = 4·5% of £2350 = 0·045 × 2350 = £105·75.

Gross Pay = £325·40 + £26 + £50 + £105·75 = £507·15.

Pension = 3% of £507·15 = 0·03 × 507·15 = 15·2145 = £15·21
Deductions = £45·30 + £75·20 + £15·21 = £135·71

Net Pay = Gross Pay – Total Deductions = £507·15 – £135·71 = £371·44

BASIC SKILL EXAMPLE 1: cost of a loan

The Carlyle family borrowed £5000 with loan protection from the Scottish Bank over a period of 36 months.
Their monthly repayment is £176·39. Calculate the cost of the loan to the Carlyle family.

Solution

The total amount they had to repay over 36 months was £176·39 × 36 = £6350·04.

They paid back £6350·04 in total, and they originally borrowed £5000. Therefore, the cost of the loan is £6350·04 – £5000 = £1350·04.

In other examples, you will have to calculate the monthly repayment yourself using the interest rate. These examples are most likely to be based on simple interest (as opposed to compound interest).

BASIC SKILL EXAMPLE 2: calculating monthly repayments

Liam takes out a £5000 loan with a simple interest rate of 11·2% per annum.
He chooses to pay the loan back over 2 years.
Calculate Liam's monthly repayment.

Solution

The interest for one year = 11·2% of £5000 = 0·112 × 5000 = £560.
The interest for two years = 560 × 2 = £1120
The total amount repayable = the original amount + the interest
= 5000 + 1120 = £6120

2 years is 24 months, so the monthly repayment is 6120 ÷ 24 = £255.

When you borrow money on a **credit card** (or a store card), you do not have to pay it all back at once. Instead you can choose how much you pay back and when. You can pay the entire balance off at once if you want, but you can pay a lot less if you want to so long as you pay at least the minimum payment set down by the company.

Definition: the **balance owed** on a credit card statement is how much money you owe the company at the current date.
Definition: the **Annual Percentage Rate (APR)** is the interest rate that you pay on your balance each year. By law, the APR must be stated for all credit cards, store cards and loans.

The Annual Percentage Rate is based on compound interest and is not equal to the monthly rate multiplied by 12 because it not based on simple interest.

BASIC SKILL EXAMPLE 3: calculate the APR

A credit card charges a monthly interest rate of 2%. Calculate the APR.

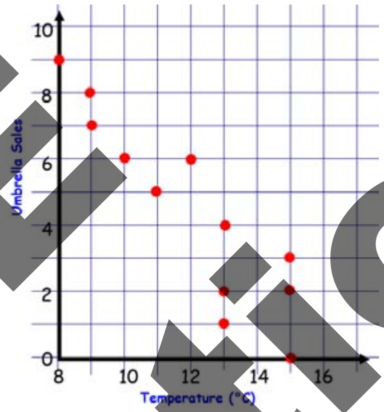
Solution

The multiplier for a monthly interest rate of 2% is 1·02.
The multiplier for compound interest for a year (12 months) is given by:
$1{\cdot}02^{12}$ = 1·268…., which is the multiplier for a 26·8% interest rate, so the APR is 26·8%.

You will *always* be asked to draw the line of best fit in a scatter graph question in an assessment question. Once you have drawn the line, you will always be asked to *use it*.

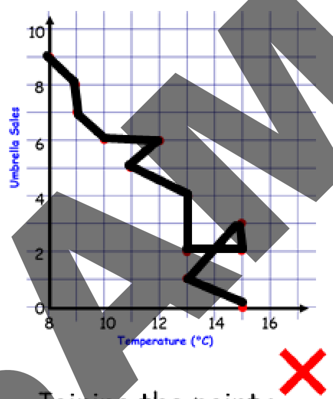
BASIC SKILL EXAMPLE 2: Drawing a Line of Best Fit

Draw a line of best fit on this scatter graph (this is the graph from the previous example)

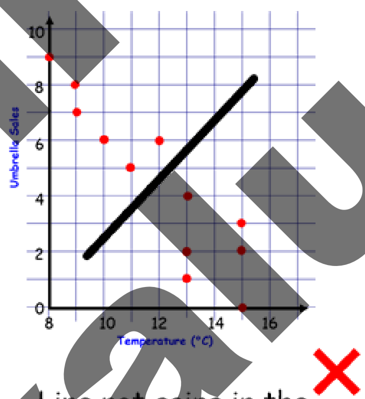


Solution

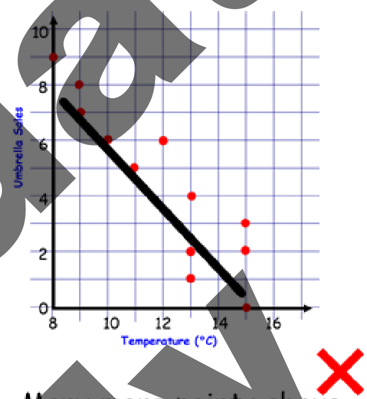
These three lines of best fit would be marked **wrong**



Joining the points. WRONG

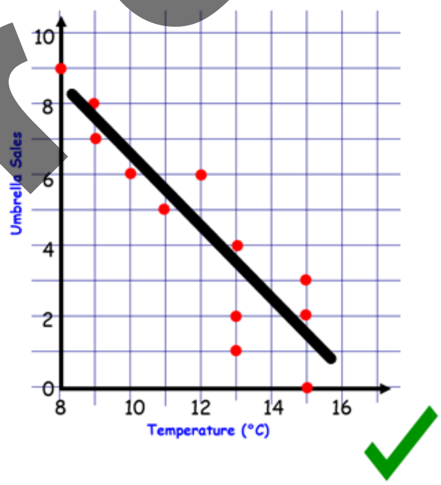


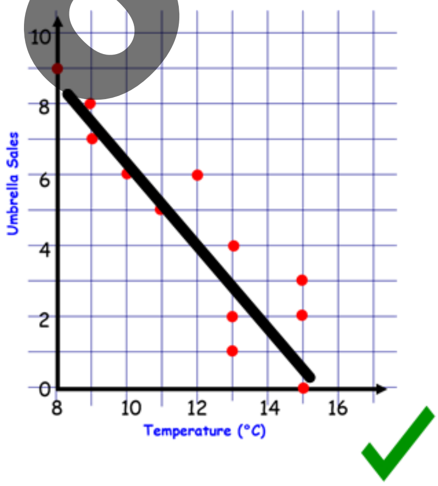
Line not going in the same direction as the points. WRONG.



Many more points above the line than below the line: too low. WRONG

Any of these answers would be **acceptable** as they are in the correct direction and have roughly the same number of points above and below the line.





BASIC SKILL EXAMPLE 2: Drawing a Box Plot

Construct a boxplot for the following set of data:

2 3 4 4 4 5 5 6 7 8 9

Solution

Step One: write down the lowest and highest values.

Lowest = 2. Highest = 9

Step Two: calculate the median. This list is already in order.

2 3 4 4 4 5 5 6 7 8 9

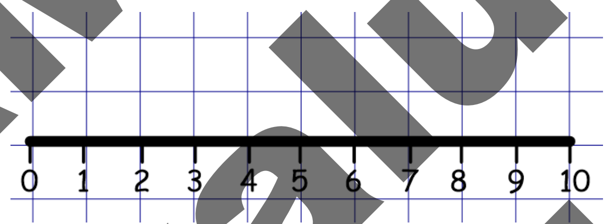
The median is 5.

Step Three: calculate the upper and lower quartiles using the method from the previous example.

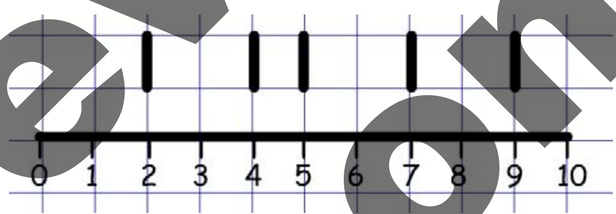
2 3 4 4 4 5 5 6 7 8 9

The lower quartile is 4 and the upper quartile is 7.

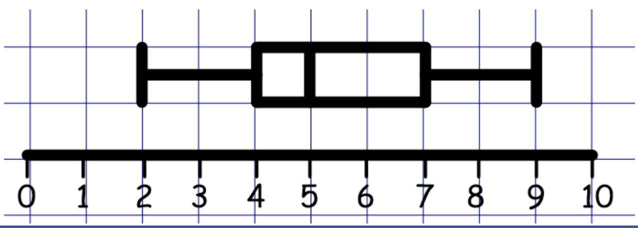
Step Four: draw and label a horizontal axis.



Step Five: draw five vertical lines corresponding to the five numbers calculated in steps 1-3. (the lowest, lower quartile, median, upper quartile and highest).



Step Six: join the middle three lines together to create a rectangle, and join the end points to create the full box plot shape.



Formula: given on the formula sheet in National 5 assessments

$$\text{standard deviation} = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} \quad \text{or} \quad \sqrt{\frac{\sum x^2 - \left(\sum x\right)^2/n}{n-1}}$$

The following symbols are used in these formulae:

- n stands for 'how many numbers are in the list'
- $\bar{x}$ stands for 'the mean' ($\bar{x}$ is read out loud as 'x bar')
- Σ means "add together" (Σ is **sigma**, the Greek capital 'S')

You only need to know how to use one of these formulae. In general, it is more helpful to just know the method rather than memorising the formula. The following two examples show how the same question is done using each method.

BASIC SKILL EXAMPLE 1a: Standard Deviation using the formula $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Calculate the mean and standard deviation of this data set: 2 3 9 6 5

Solution

Step 1: Calculate the Mean.
There are five numbers, so $n = 5$.

Mean: $\frac{2+3+9+6+5}{5} = \frac{25}{5} = 5$, so the mean is 5.

Step 2: Draw up a table with column headings x, $x-\bar{x}$ and $(x-\bar{x})^2$.

x	$x-\bar{x}$	$(x-\bar{x})^2$
2		
3		
9		
6		
5		

Step 3: Complete the table, remembering that $\bar{x} = \text{the mean} = 5$.

- *In the middle column, take away the mean from each number in the left-hand column.*
- *In the right-hand column, square each number in the middle column.*

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	–3	9
3	–2	4
9	4	16
6	1	1
5	0	0
	TOTAL	**30**

Step 4: find the total of the final column
In this example, $\sum(x-\bar{x})^2 = 30$.

Step 5: use the formula, remembering that $n = 5$ because there were five numbers.

(continued on next page)

(Basic Skill Example 1a continued)

$$s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$=\sqrt{\frac{30}{5-1}} \quad =\sqrt{\frac{30}{4}}$$

$$=\underline{\underline{2\cdot74}} \text{ (2 d.p.)}$$

If you don't like using the formula, you can just remember these two steps:

- Divide by $n - 1$.
- Square root.

The next example is the same question as in the previous example, but using the other formula.

BASIC SKILL EXAMPLE 1b: Standard Dev. using the formula $s=\sqrt{\frac{\sum x^2-\frac{(\sum x)^2}{n}}{n-1}}$

Calculate the mean and standard deviation of this data set: 2 3 9 6 5

Solution

Step 1: Calculate the Mean.
There are five numbers, so $n = 5$.

Mean: $\frac{2+3+9+6+5}{5}=\frac{25}{5}=5$, so the mean is 5.

Step 2: Draw up a table with column headings x and x^2.

x	x^2
2	
3	
9	
6	
5	

Step 3: Complete the table.

	x	x^2
	2	4
	3	9
	9	81
	6	36
	5	25
TOTAL	**25**	**155**

Step 4: Obtain the totals from the table.
In this example, $\sum x=25$, $\sum x^2=155$.

Step 5: use the formula, remembering that $n = 5$ because there were 5 numbers.

$$s=\sqrt{\frac{\sum x^2-\frac{(\sum x)^2}{n}}{n-1}}$$

$$=\sqrt{\frac{155-\frac{25^2}{5}}{5-1}}$$

$$=\sqrt{\frac{155-125}{4}}=\sqrt{\frac{30}{4}}$$

$$=\underline{\underline{2\cdot74}} \text{ (2 d.p.)}$$

What should an exam question look like?

In assessments for National 5 Applications, standard deviation questions should always:

- Ask you to work out the standard deviation for a list of (probably 7 or fewer) numbers.

You **might** also have to:

- Write a comment about what the mean and/or standard deviation show us (see page 96).

(There are many ways questions may be adapted and so this list can never cover everything).

Drawing Pie Charts

Interpreting a pie chart was covered in the notes for the *Numeracy* unit on page 30. As part of the *Managing Finance and Statistics* unit, you may be required to draw a pie chart.

You need to be able to work out the angles you would use to draw the slices in a pie chart. You have to work out what fraction of the circle each slice has to be.

BASIC SKILL EXAMPLE: Calculating the Angles in a Pie Chart

In a school survey 200 pupils were asked what their favourite takeaway food was. The results were:

Food	Frequency
Pizza	55
Fish and Chips	64
Chinese	81

Calculate the angle required for each sector to represent this data on a pie chart.

Solution

Step One: Work out the **fraction** for each slice

There were 200 pupils in total, so the fraction has to be out of 200

Pizza: $\frac{55}{200}$ Fish and chips: $\frac{64}{200}$ Chinese: $\frac{81}{200}$

Step Two: work out each slice as a fraction of 360°

Pizza: $\frac{55}{200}$ of $360° = \underline{99°}$ $(360 \div 200 \times 55)$

Fish and chips: $\frac{64}{200}$ of $360° = \underline{115 \cdot 2°}$ $(360 \div 200 \times 64)$

Chinese: $\frac{81}{200}$ of $360° = \underline{145 \cdot 8°}$ $(360 \div 200 \times 81)$

Step Three: double check that your answer adds up to 360°

(in an exam you may also be expected to draw the pie chart. To do this, you must ensure each sector is labelled, and all your angles must be drawn to a tolerance of ±1°).

Index of Key Words

All information in this revision guide has been prepared in best faith, with thorough reference to the documents provided by the SQA, including the course arrangements, course and unit support notes, exam specification, specimen question paper and unit assessments.

These notes will be updated as and when new information becomes available.

We try our hardest to ensure these notes are accurate, but despite our best efforts, mistakes sometimes appear. If you discover any mistakes in these notes, please email us at david@dynamicmaths.co.uk.
An updated copy of the notes will be provided free of charge!
We would like to hear any suggestions you may have for improving our notes.

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Version 1.0: Published October 2014.

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