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National 4 Mathematics Revision Notes

Last updated July 201[illegible]

Use this booklet to practise working independen[illegible] you [illegible] have to in course assessments and the Value Added Unit (AVU).

- Get in the habit of turning to this [illegible]klet to refres[illegible]ur memory.
- If you have forgotten how to do a[illegible]d, **exa**[illegible]**les** are given.
- If you have forgotten what a[illegible]d m[illegible]e **index** (back p[illegible]e) to l[illegible] up.

As you get closer to the fi[illegible]st, y[illegible] should [illegible]m to use this bookle[illegible]d less.

This booklet is for:

- Students [illegible]the[illegible]tio[illegible] Mathematics course[illegible]
- Students [illegible] more of the National 4 ma[illegible]tics units: **Numeracy**, **Expressions** [illegible]**l Fo**[illegible]**ulae** or **Relati**[illegible]**ship**[illegible]

Thi[illegible]**tains:**

- T[illegible]no[illegible]tant facts you need t[illegible]emo[illegible]for National 4 Mathem[illegible]cs.
- Ex[illegible]s that take you through t[illegible] m[illegible]ommon **routine** questions in [illegible]h topi[illegible]
- [illegible]finitions of the key wor[illegible] you n[illegible] know.

[illegible]**this booklet:**

- To refresh your memory [illegible]ethod you were taug[illegible] when you are stuck on a hom[illegible]uestion or a practice test qu[illegible]n.
- To memorise [illegible]ey f[illegible]ts[illegible]hen revising for assessments[illegible]the Value Added Unit.

The key to revising [illegible]maths test is to do qu[illegible]ons, [illegible] to read notes. **As well as using t[illegible] booklet, you should also:**

- P[illegible]e by working through exercises on topi[illegible]u need more practice on – such as [illegible]ion booklets, textbooks, websites, or exercises suggested by your teacher.
- Wor[illegible]ough practice tests.
- [illegible]k your teacher when you come across a question you cannot answer.
- [illegible]e resources online (a link that can be scanned with a SmartPhone is on the last page).

Contents

Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

Formulae that <u>are given</u> on the formula sheet in the Added Value Unit (or in unit assessments)

Topic	Formula(e)	Page Reference
Circumference of a circle	$C = \pi d$	See page 37
Area of a circle	$A = \pi r^2$	See page 37
Curved surface area of a cylinder	$A = 2\pi rh$	See page [illegible]
Volume of a cylinder	$V = \pi r^2 h$	See page [illegible]
Volume of a prism	$V = Ah$	See page 42
Gradient	$\text{Gradient} = \dfrac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 35
Pythagoras' Theorem	$a^2 + b^2 = c^2$	See page 61
Trigonometry in a right angled triangle	$\sin x° = \dfrac{\text{Opp}}{\text{Hyp}}$ $\cos x° = \dfrac{\text{Adj}}{\text{Hyp}}$ $\tan x° = \dfrac{\text{Opp}}{\text{Adj}}$	See page 63

Formulae that are <u>not</u> given in the Added Value Unit (or in unit assessments)

Topic	Formula(e)	Page Reference
Area of a [illegible]	$A = LB$	See page 19
Area of a square	$A = L^2$	See page 19
Area of a triangle	$A = \dfrac{BH}{2}$	See page 19
Volume of a cuboid	$V = LBH$	See page 19
Percentage increase and decrease	$\dfrac{\text{change}}{\text{original amount}} \times 100$	See page 18
Range	$\text{Range} = \text{Highest} - \text{Lowest}$	See page 49
Mean	$\text{Mean} = \dfrac{\text{Total}}{\text{How many}}$	See page 49
Equation of a straight line	$y = mx + c$	See page 56
Speed, Distance, Time	$S = \dfrac{D}{T}$ $T = \dfrac{D}{S}$ $D = ST$	See page 22

Add, Subtract, Multiply, Divide and Rounding

Written Sums

You are expected to be able to do written add, take away, multiply and divide sums. You need to know how to do 'carrying' and (in subtraction sums) 'borrowing' to complete these sums.

Examples 1 – whole numbers

Calculate: (a) 5629 + 3783, (b) 4007 – [illegible] (c) [illegible] × 7, (d) 3875 ÷ 5

Solutions

Worked solutions (with carry[illegible] [illegible] borro[illegible]g as required) are sho[illegible] bel[illegible]

(a)	(b)	(c)	(d)
5629 + 378[illegible] = [illegible]41[illegible]	[illegible]0[illegible]7 − [illegible]78 = [illegible]329	[illegible]38 × 7 = 3066	[illegible]77[illegible] = 5)[illegible]75

For the Added [illegible] Un[illegible] you need to be [illegible]ble t[illegible] [illegible] thes[illegible] [illegible]hods with decimals.

For [illegible] may be asked to add and ta[illegible] [illegible]ay [illegible]mals without a calculator.

Exampl[illegible] – add and then take awa[illegible]

T[illegible] make a fruit drink [illegible] fath[illegible] [illegible] mi[illegible]es 2·75 litres of [illegible] and [illegible] [illegible]res of juice. 1·48 litres o[illegible] [illegible]e ju[illegible] drunk. How much [illegible]?

[illegible]olution

We add 2·7[illegible] [illegible] 375, and then we take away [illegible] fro[illegible] [illegible] answer.

Step one[illegible] [illegible]g

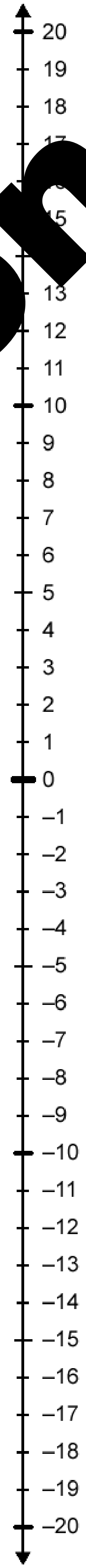
$$\begin{array}{r} 2{\cdot}750 \\ +\,0{\cdot}375 \\ \hline 3{\cdot}125 \end{array}$$

Answer to step 1: 3·125 litres.

Step two: take away

$$\begin{array}{r} \cancel{3}{\cdot}\cancel{1}25 \\ -\,1{\cdot}480 \\ \hline 1{\cdot}645 \end{array}$$

Answer: There is 1·645 litres of juice left. **(in a real life situation we should always give our answer in a sentence, and units are essential)**



<u>Examples 1</u>

4652 rounded to the nearest ten is: 4650
4652 rounded to the nearest hundred is: 4700
4652 rounded to the nearest thousand is: 5000

23·6666666 rounded to one decimal place is: 23·7
23·6666666 rounded to two decimal places i[illegible] 23·67
£23·6666666 rounded to the nearest penn[illegible] £23·67

To round to **two decimal places**, we look at th[illegible]git af[illegible] the point (the thousandths):

- If this digit is 0, 1, 2, 3 or 4, we keep the seco[illegible] (the hundredths) the same.
- If this digit is 5, 6, 7, 8 or 9, we [illegible] the [illegible]nd digit up.

<u>Examples 2 – rounding to [illegible]l places</u>

Round 75·68[illegible]nd 5[illegible] to two decimal places.

Solutions

In 75[illegible]he t[illegible]d [illegible] after the point is 2. T[illegible]efore [illegible]ep the second digit [illegible]am[illegible]he answer is **<u>75·68</u>**.

[illegible]57·249, [illegible]hird digit after the [illegible] is 9[illegible]ore we put the second digit a[illegible]point up from 4 to 5 and the[illegible]er is [illegible]25.

Roundi[illegible] the nearest **significan[illegible]gu[illegible]** [illegible] different thin[illegible] de[illegible]ding [illegible] the [illegible] th[illegible] number we are roun[illegible]ng. I[illegible]e [illegible] rounding a num[illegible] with [illegible] be[illegible] the decimal point, it me[illegible] to r[illegible] to the nearest thousa[illegible] 3745 r[illegible]ded t[illegible]e significant figure i[illegible]000). [illegible] we are rounding a number with [illegible] digits [illegible]ore [illegible] decimal point, it means [illegible] to the nearest ten (i.e. [illegible]unde[illegible] to one significant figure is [illegible]

Adding and T[illegible]ng [illegible]way Negative Nu[illegible] (In[illegible]gers)

Defini[illegible]: an **integer** is any positive or negat[illegible] number, including zero. You nee[illegible] able to use integers in everyday situati[illegible] (e.g. temperature), and in sums.

[illegible]g a[illegible] taking away integers is all to do with moving up and down a number line. In a[illegible]xam, you could draw your own number line to count up and down on if it helps yo[illegible]

<u>Example 1 – temperature</u>

The temperature at midnight was –6°C. By midday it had risen by 23°C. What was the temperature at midday?

Solution

Start at –6°C. Move <u>up</u> the number line by 23. Answer: **<u>17°C</u>**.

Fractions and Percentages

Changing Fractions, Decimals and Percentages

To change a **fraction to a decimal**, divide the top number by the bottom number.

Example 1 – fractions to decimals

Change the fraction $\frac{13}{20}$ to a decimal [illegible]

Solution $13 \div 20 = 0\cdot65$ [illegible]**r:** $0\cdot65$

To change a **percentage to a decimal**, [illegible] by 1[illegible]
To change a **decimal to a perc**[illegible], m[illegible] 100.

Example 2 – percentages a[illegible]s

Change the p[illegible]enta[illegible]% to a decimal.
Change the[illegible]l 0·8[illegible]percentage.

Solution

(a) 2[illegible]25, **Answer:** [illegible] = [illegible]025 as a decimal.
(b) 0·8 × [illegible] = 8[illegible] **Ans[illegible]:** 0·8[illegible]% as a percentage.

To [illegible]entage to a fraction, the num[illegible] on th[illegible]p is always 100 and the numb[illegible] on t[illegible]om is the number bef[illegible] the p[illegible]entage sign.

Exampl[illegible] percentages to fractio[illegible]

[illegible]rite the percentage [illegible]% as [illegible]fra[illegible]tion.

So[illegible]on 37% = [illegible] 10[illegible]

To change a decim[illegible]action, the denominator (th[illegible]mber[illegible]e bottom) depends on how [illegible]ny[illegible]gi[illegible] there are after the decimal p[illegible] If there:

- is **one dig**[illegible]e point, the denomin[illegible]or i[illegible] (e.g. $0\cdot3=\frac{3}{10}$).
- [illegible]e **two digits** after the point, the denom[illegible]s 100 (e.g. $0\cdot17=\frac{17}{100}$).
- a[illegible]**ree digits** after the point, the denominator is 1000 (e.g. $0\cdot451=\frac{451}{1000}$).

[illegible]ple 4 – decimals to fractions

Change the decimal 0·013 to a fraction
Change the decimal 0·57 to a fraction

Solution

(a) $0\cdot013=\frac{13}{1000}$ (b) $0\cdot57=\frac{57}{100}$

To change a fraction to the percentage, there are two steps:

1. Change to a decimal by **dividing**
2. Change to a percentage by **multiplying by 100**

<u>Example 5 – fractions to percentages</u>

Pete got 24 out of 32 for an exam. What is his mark as a percentage?

Solution

As a fraction, Pete got $\frac{24}{32}$.

To change this to a decimal, divide: [illegible] ÷ 32 = 0·75

To change 0·75 to a percentage, multiply by [illegible] 0·75 × 100 = **<u>75%</u>**

A quick way of rememberi[illegible] is t[illegible] top ÷ bottom × 100

Calculating Fraction[illegible]

To calculate a fracti[illegible] divi[illegible] the number on the bott[illegible] (t[illegible]nominator) and multiply by the [illegible] the [illegible]p (the numerator)

Some[illegible] this as **'Divide by the botto[illegible] times by the top'**

<u>Exa[illegible]</u>

A w[illegible]machine costs £385. $\frac{3}{5}$ o[illegible] cost is for the material[illegible] What [illegible] cost of the materials?

[illegible]on

$\frac{3}{5}$ of 385 = 385 ÷ 5 × 3

= 77 × 3

[illegible]

Percen[illegible]ages wi[illegible] a calculator

You [illegible]ld <u>know</u> the following:

[illegible]ercentage	Fraction	Percentage	Fraction
50%	$\frac{1}{2}$	10%	$\frac{1}{10}$
25%	$\frac{1}{4}$	1%	$\frac{1}{100}$
75%	$\frac{3}{4}$	$33\frac{1}{3}\%$	$\frac{1}{3}$
20%	$\frac{1}{5}$	$66\frac{2}{3}\%$	$\frac{2}{3}$

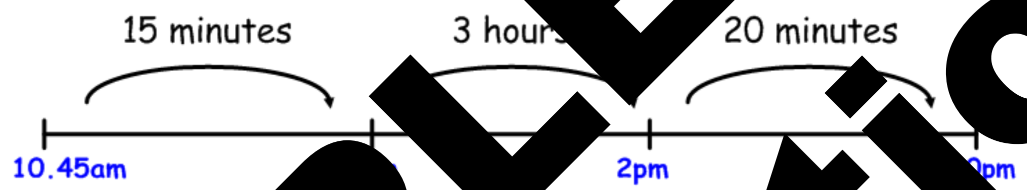
Speed, Distance and Time

You need to know how to work out how much time an event takes from beginning to end. The best way is to split each question up into smaller steps:

Example 1

How long is it from 10:45am to 2:20pm?

Solution

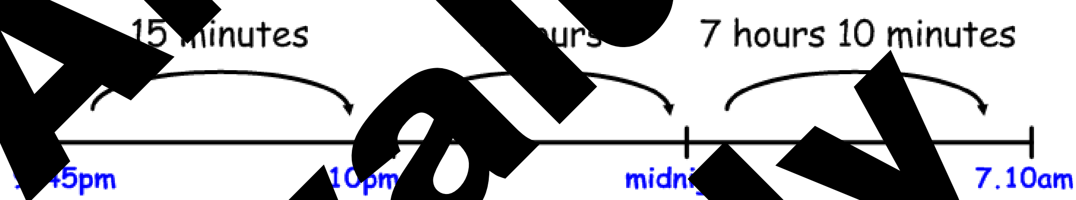


Total time = 15 minutes + [illegible] 20 minutes = **3 hours 35 minutes** ([illegible] 3:35)

Example 2

A plan[illegible] 9:45pm and arrives in Los A[illegible] the next mor[illegible]. [illegible]ow long was the flight[illegible]

Solut[illegible]



T[illegible]l time = 15 min + 2 [illegible] hours + 10 min = **9 hour[illegible]** [illegible]

Changing tim[illegible] d[illegible]imal

Remem[illegible]er basic fractions and decimals
¼ = 0·25 ½ = [illegible] ¾ = 0·75

[illegible]efo[illegible]ours 15 minutes is not entered into the calculator as 2·15 hours. Instead it [illegible]25 hours:

1 hour 30 minutes	= 1 ½ hours	= 1·5 hours
5 hours 15 minutes	= 5 ¼ hours	= 5·25 hours
45 minutes	= ¾ hour	= 0·75 hour

We can change other fractions of an hour using our knowledge of decimals and tenths. In particular, it is worth noting the fact that **0·1 hour = 6 minutes**.

Example 1
Change 7·2 hours into hours and minutes.

Solution
0·2 hours × 60 = 12 minutes

Answer: 7·2 hours = 7 hours 12 minutes.

Example 2
Change 5 hours 18 minutes into a decimal

Solution
18 minutes is $\frac{18}{60}$ as a fraction

If we simplify this fraction we get $\frac{18^{\div 6}}{60_{\div 6}} = \frac{3}{10}$

As a decimal, $\frac{3}{10}$ is written 0·3

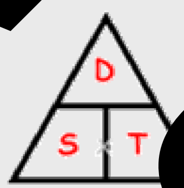
Answer: 5 hours 18 minutes = **5·3 hours**

Speed, Distance and Time Calculations

Formulae: These formulae are **not** given on the formula sheet in assessments

Speed = $\frac{\text{Distance}}{\text{Time}}$ Time = $\frac{\text{Distance}}{\text{Speed}}$ Distance = Speed × Time

If you have been taught it, you might also like to use the Speed Distance and Time triangle



Units: The units for speed, distance and time are interlinked:
- If distance is in **km** and time is in **hours** (h), speed is measured in **km/hour**
- If distance is in **metres** (m) and time is in **seconds** (s), speed is measured in **m/s**
- If the speed is in **cm/min**, then the distance is in **cm** and the time in **minutes**

Sometimes speed is referred to as **average speed** or **mean speed** to reflect the fact that it can vary during a journey. This makes no difference to how you answer a question.

Example 1 – find speed

I drive 90km in 2 hours and 15 minutes. Calculate my average speed.

Solution

We are working out speed, so we use the formula $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$.

2 hours and 15 minutes is not 2·15 hours. It is 2 ¼ hours = 2·25 hours.

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}} \\ &= \frac{90}{2\cdot 25} \\ &= \underline{40\text{km/h}}\end{aligned}$$

Example 2 – find distance

A bird flies 3½ hours [illegible] average speed of 42km/h. [illegible] far [illegible]s it fly?

Solution

We a[illegible]king [illegible]t [illegible]ance, so we use the for[illegible]la D[illegible] = Speed × Time

3 ½ ho[illegible]s [illegible]0 hours. It is 3·5 hours.

$$\begin{aligned}\text{Dis[illegible]} &= \text{Speed} \times \text{Time} \\ &= 42 \times 3\cdot 5 \\ &= \underline{147\text{km}}\end{aligned}$$

Ex[illegible]ple 3 – find time

A driver travels 1[illegible]9 mi[illegible] at an average speed of 3[illegible]mph. [illegible] lon[illegible]oes it take her? Give y[illegible]wer in hours and mi[illegible]

Solution

We are w[illegible] time, so we use the [illegible] Tim[illegible] = $\dfrac{\text{Distance}}{\text{Speed}}$

$$\begin{aligned}\text{Time} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{129}{30} \\ &= 4\cdot 3\end{aligned}$$

4·3 hours is not 4 hours 3 minutes.

0·3 × 60 = 18, so 4·3 hours = **4 hours 18 minutes**.

Expressions and Formulae Unit

Algebra

Using a Formula

Read the question carefully and substitute in the numbers you are given.

Definition: Evaluate means "do the sum"

Example 1

Evaluate 2*a* – 3*c* when *a* = 12 and *c* = 1·5

Solution

$2a - 3c$

$= 2\times 12 - 3\times 1\cdot 5$

$= 24 - 4\cdot 5$

$= \underline{19\cdot 5}$

Example 2

Evaluate 3*x*² when *x* = 5

Solution

$3x^2$

$= 3\times 5^2$

$= 3\times 25$ (NOT 15² as squaring comes before multiplying in BODMAS)

$= \underline{75}$

Example 3

***S* = 3*bc* – *a*. Evaluate *S* when *a* = 10, *b* = 2 and *c* = 7**

Solution

$S = 3bc - a$

$= 3\times 2\times 7 - 10$

$= 42 - 10$

$= \underline{32}$

You would also be expected to understand a formula in a real-life situation, where the letters would be explained to you.

Example 4 – real life situation

The cost of hiring a car is given by the formula $C = 25d + 2p$, where *C* is the cost of hiring the car, *d* is the number of days hired for, and *p* is the number of litres of petrol used.

explicitly told to. *Only* numbers inside the bracket are multiplied. Anything else that is not inside the bracket should remain unchanged until you start simplifying.

<u>Examples 2</u>

Multiply out the brackets and simplify:

$\mathbf{4(m+5)-18}$ $\mathbf{4(x+5)+3(x-2)}$

Solution

$4(m+5)-18$
$=4m+20-18$
$=\underline{4m+2}$

$4(x$ [illegible] $)+3(x-2)$
[illegible] $+2$ [illegible] $3x-6$
[illegible]

However, be careful the <u>only</u> number [illegible] etters) t [illegible] you multiply by ar [illegible] th [illegible] are right next to the bracket. An [illegible] ing [illegible] t inside the bra [illegible] ket sho [illegible] remain unchanged until you [illegible] lifyi [illegible]

<u>Examples 3</u>

Multiply ou [illegible] **brac** [illegible] **nd simplify:**

$\mathbf{4+7(a+2)}$ $\mathbf{2x+3}$ [illegible]

Solution

$4+7(a$ [illegible]
[illegible] $4+7a+1$ [illegible] (<u>NOT</u> $11(a+2)$)
$=$ [illegible] $7a+18)$

$2x+$ [illegible] $1)$
[illegible] $3x+3$ (<u>NOT</u> $5x(x+1)$)
$=$ [illegible] $x+3$ (or $3+5x$)

Facto [illegible] g

De [illegible] **tion: Factorise** means "p [illegible] the b [illegible] kets back in". Think o [illegible] the opp [illegible] e of [illegible] plying out the brack [illegible]

<u>Example 1</u>

Factoris [illegible] $\mathbf{a}$ [illegible] $\mathbf{b}$ **Factorise 1** [illegible] $\mathbf{5y}$

Solution

[illegible] e highest common factor is 3
[illegible] rite 3 in front of the bracket
[illegible])

T [illegible] common factor is 5
Write [illegible] in front of the brackets
5( )

[illegible] Work out what goes inside the brackets (it may help to think of dividing)

Answer: $3(2a+3b)$ **Answer:** $5(3x+5y)$

You always need to take the ***largest possible*** number (and/or letter) outside the brackets. You can spot these questions as they will say **factorise fully** instead of just **factorise**.

<u>Example 2</u>

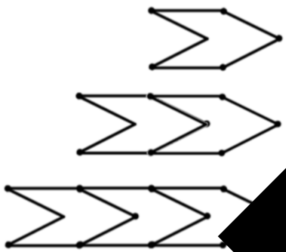
Factorise fully: $\mathbf{18x+24}$

In National 4 assessments, you are expected to work with number patterns from diagrams. Each question will be set in a different 'real-life' situation, but the basic structure of the question tends to be the same.

Part (a): complete a table with numbers obtained from the diagram.
Part (b): write a formula in using the method outlined above.
Part (c): use the formula to go backwards.

<u>Example 2 – 2011 Standard Grade General Past Exam Question</u>

Margaret is working on the design for a gold bracelet. She is using gold lengths to make each section.



Complete the table b

Number of sections (s)	1	2				10
Number of gold lengths (g)	6	10				

Write down a formula for calculating the number of gold lengths (g) when you know the number of sections

Margaret has 66 gold lengths to make a bracelet. How many sections does her bracelet contain?

Solution

The completed table is:

Number of sections (s)	1	2	3	4		10
Number of gold lengths (g)	6	10	14	18		42

(b) Using the same method as in example 1:
<u>Step one</u> – write the letters the correct way around: $g = ?s + ?$
<u>Step two</u> – the bottom row is going up by 4, so we must multiply by 4:
$g = 4s + ?$
<u>Step three</u> – the 4 times table is 4, 8, 12, … We have 6, 10, 14, … so we are adding 2 each time **<u>Final Answer:</u>** $g = 4s + 2$

(c) There are a few methods to do this part. One way is to add more columns to the table until you get to 66 on the bottom row. This would work, but it takes a while. Another way is to solve an equation. **However whatever method you use, you must show some working.**

To solve an equation for 66 gold lengths, we take our formula from part (b) ($4s + 2$) and write '= 66' on the end of it.

$$4s+2=66$$
$$4s=66-2 \qquad \text{(moving the } +2 \text{ over to become } -2\text{)}$$
$$4s=64 \qquad \text{(simplifying } 66-2\text{)}$$
$$s=\frac{64}{4} \qquad \text{(dividing by 4)}$$
$$s=16$$

Gradient

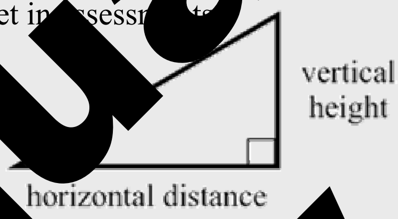
The **gradient** of a slope is its steepness. The higher the gradient is, the steeper the slope is.

- A gradient of 2 means 'for every 1 you go along, you go up 2'.
- A gradient of $0{\cdot}68$ means 'for every 1 you go along, you go up $0{\cdot}68$'.

Sometimes gradient is referred to as **average gradient** or **mean gradient** to reflect the fact that it can vary as you go up a slope. This makes no difference to how you answer a question.

Formula: this formula is given on the formula sheet in assessments.

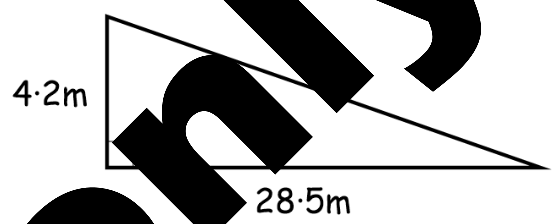
$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$



Example

(a) Calculate the gradient of this wheelchair ramp.

(b) Regulations say that the gradient of a wheelchair ramp must be less than $0{\cdot}1$. Does this ramp meet the regulations? Explain your answer.



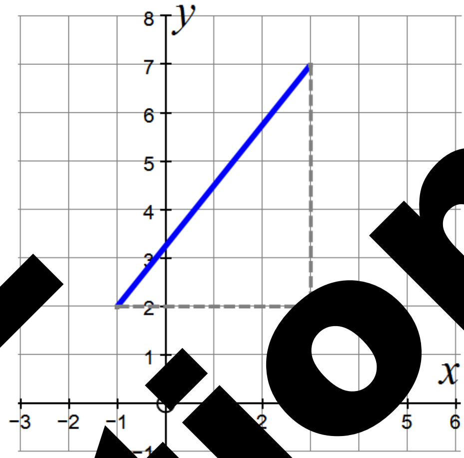
Solution

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$
$$= \frac{4\cdot 2}{28\cdot 5}$$
$$= \underline{\underline{0\cdot 147}} \text{ (3d.p.)}$$

b) No, the ramp does not meet regulations as $0{\cdot}147$ is bigger than $0{\cdot}1$.

Example 2 – from coordinates

Find the gradient between the coordinate points (–1, 2) and (3, 7)



Solution

From counting squares in the diagram, the vertical height is 5 and the horizontal distance is 4.

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$
$$= \frac{5}{4}$$
$$= \underline{\underline{1 \cdot 25}}$$

In some situations, such a[illegible]oor[illegible]ate grids, direction is importa[illegible]hese situations, a **positive** [illegible]dien[illegible]gradients such as 2, 10 or $\frac{1}{[illegible]}$[illegible]ns th[illegible]e slopes **upwards**. A **n**egativ[illegible]ra[illegible]t (e.g. gradients such as –2, –[illegible]0 [illegible] –[illegible]) means the line slopes **dow**[illegible]

Definition: a **composite shape** is one made by joining two or more other shapes together. In the exam, areas will always be of composite shapes, usually made up of rectangles, squares, triangles or semi-circles (for semicircles see page 37).

The method to work out the area of a composite shape is always the same:

Step one – split the shape up into smaller, simpler shapes.
Step two – work out the area of each smaller shape separately.
Step three – either add or take away the areas:

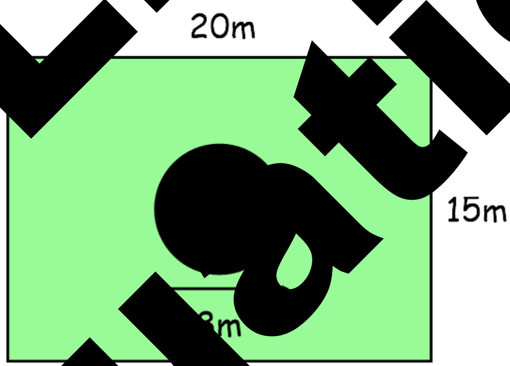
- If the two shapes are joined together, you **add** the areas.
- If one shape is cut out of the other, you **take away** its area.

Example 1

The diagram shows a grass lawn. The lawn is in the shape of a rectangle measuring 15 metres by 20 metres.

There is a circular flowerbed cut out of the middle of the lawn with diameter 8 metres.

Calculate the area of the grass.



Solution:

Area of rectangle:

$A = LB$

$= 20 \times 15$

$= 300\text{m}^2$

Area of circle:

Diameter of circle is 8m so radius is 4m

$A = \pi r^2$

$= \pi \times 4^2$

$= 50 \cdot 265...\text{m}^2$

Total area remaining = $300 - 50 \cdot 265 = 249 \cdot 73\text{m}^2$

You need to be able to work out the area of a **trapezium**, a **parallelogram** or a **kite** by splitting the shape up into rectangles and triangles.

Definitions:

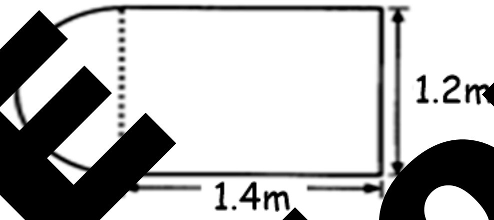
- A **quadrilateral** is a shape with four straight sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides. It can be split up to be a rectangle and two identical right-angled triangles.
- A **kite** is a quadrilateral with one line of symmetry. It can be split up to be four triangles (two pairs of equal triangles).
- A **trapezium** is a quadrilateral with one pair of parallel sides. It can be split up to be a rectangle and one or two right-angled triangles.

Step three – the two shapes are joined together, so **add** the areas

Total Area = 54 + 162 = **216cm²**

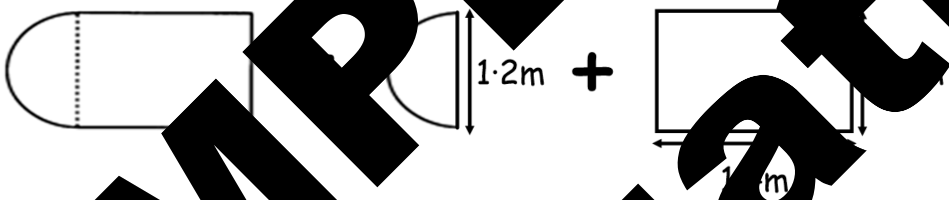
Example 3 (Standard Grade General paper 2001)

The base of a lift is in the shape of a rectangle with a semicircle end as shown. Calculate the area of the base of the lift.
Round your answer to 1 decimal place



Solution

Step one – start by splitting th[illegible] up i[illegible] rectangle and a sem[illegible]le.



Step [illegible] [illegible]te [illegible] area of each shape.

1[illegible] the [illegible]ameter of the c[illegible]e, s[illegible] is th[illegible]dius.

[illegible] of semicircle $= \frac{\pi r^2}{[illegible]}$

$= \frac{[illegible] \times 0[illegible]}{[illegible]}$

$= 0 \cdot [illegible]...\text{m}^2$

Area of rectangle $= LB$[illegible]

$= 1 \cdot 68$[illegible]

Step three – [illegible] two s[illegible] are joined together, [illegible] areas

T[illegible] A[illegible]ea [illegible] 1·68 + 0·565…. = 2·245[illegible] = [illegible]m² (1 d.p.)

Volum[illegible] of 3-d Shapes

Defi[illegible]: A **prism** is a 3d solid with a uniform cross-section. In everyday [illegible]uag[illegible] could say that it is the 'same shape all the way along'.

By [illegible]nition a cube and a cuboid are prisms. Volumes of cubes and cuboids are [illegible]ed in the numeracy unit on page 19.

Definition: The **cross-section** is the shape at either end (and throughout the middle) of a prism.

Formula: this formula <u>is</u> given on the formula sheet in assessments

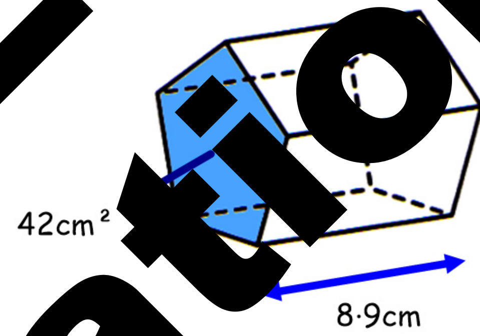
Volume of a prism: $V = \text{Area of cross-section} \times \text{height}$

$V = Ah$

In volume questions, the units are always in 'cubic' units. If all the lengths in the question are in centimetres (cm), the volume is in 'cubic centimetres' (cm^3). If all the lengths in the question are in miles, then the volume [illegible] measured in $miles^3$.

<u>Example 1 – area of cross-section is given</u>

The shape shown in the diagram on th[illegible] is a prism with length 8·9cm. The cross-section (shaded) is a pentago[illegible] with area $42cm^2$. Calculate the volum[illegible] prism[illegible]



Solution

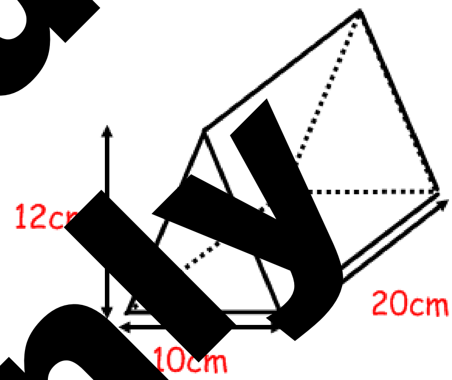
$V = Ah$

$= 42 \times 8 \cdot 9$

$= 373 \cdot 8$[illegible]

<u>Example 2</u> [illegible]

Find t[illegible] this prism, whose cr[illegible] section is [illegible]angle.



Solu[illegible]:

[illegible] of this prism is the [illegible] (triangular) end to the o[illegible]r. s[illegible]e, the length is 20cm.

<u>Step 1:</u> Work out the are[illegible] the cross-section

In this shape, the cross-[illegible]on is a triangle. The formu[illegible] area of a triangle is $\frac{1}{2}bh$ (see page[illegible])

[illegible]portant: you will use a <u>different</u> fo[illegible]la in [illegible]h question, depending on [illegible]ether the cross section is a rectangle, [illegible]riangle, circle, semicircle etc.

$$A_{triangle} = \tfrac{1}{2}bh$$
$$= 10 \times 12 \div 2$$
$$= 60cm^2$$

<u>Step 2:</u> Use the formula to find the volume

$$V = Ah$$
$$= 60 \times 20$$
$$= \underline{\underline{1200cm^3}}$$

Statistics and Probability

Grouped Frequency Tables

A **frequency table** (often commonly called a **tally chart**) is used to summarise **frequencies** (totals) for a set of data. Tally marks are not strictly compulsory, but should usually be used to avoid errors in totalling.

A **grouped frequency table** is used for spread[illegible] dat[illegible]here there are a lot of different, but close, values – e.g. heights. Rathe[illegible] havin[illegible]e number per row (which would result in a massive table), we group [illegible] into rows (e.g. 5s, 10[illegible] 100s) as appropriate.

When discrete data is displayed i[illegible] gro[illegible] fre[illegible]cy table, we wr[illegible]te rows [illegible]9, 10–19 etc. Notice that they a[illegible]itten [illegible]10–20, as it woul[illegible] clear[illegible]h row '10' would go in.

Example

The ages of [illegible]loye[illegible] a bank were record[illegible] an[illegible] a[illegible]own in the table b[illegible]w:

42	**29**	**41**	**[illegible]8**	**46**	**37**	**4[illegible]**	**[illegible]**	**38**	**22**	**30**
[illegible]	**59**	**[illegible]**	**52**	**48**	**[illegible]**	**[illegible]**	**1[illegible]**	**45**	**28**	**41**

Co[illegible] grouped frequency tabl[illegible] show this information.

Soluti[illegible]

Age	[illegible]ally	[illegible]ber of em[illegible]ees
10 – 19	|||	[illegible]
20 – 2[illegible]	||||	4
30[illegible]	~~||||~~	5
40[illegible]	~~||||~~ ||	7
50[illegible]	||	2
60 – 69	|	1
TOTAL		22

M[illegible], Mode and Range

[illegible]nition: The **range** is the difference between the highest and the lowest numbers. It shows how varied (or consistent) a list of numbers is.

Formula: this formula <u>is **not** given</u> on the formula sheet in assessments

Range = Highest – Lowest

Equations with letters on both sides

In National 4 assessments, you will have to solve an equation that has letters on both sides.

***[Before you start** (optional) – write the "invisible plus signs" in, in front of anything that does not have a sign in front of it already, to remind you it is positive.]*

- **Step one** – move everything with a letter in to the left-hand side, and all the numbers to the right-hand side, remembering to "change side and do the opposite".
- **Step two** – simplify each side.
- **Step three** – solve the resulting equation.
- **Final step** – double check your answer works by substituting it back in to both sides of the original equation to check that both sides give the same answer.

<u>Example 1</u>

Solve algebraically the equation $2a+5=15-2a$

Solution

Optional first step *– write the "invisible plus signs" in front of anything that does not already have a sign.*

Step one – move the '–2a' over to the left-hand side where it becomes '+2a'. Move '+5' to the right-hand side where it becomes '–5'.

Step two – simplify both sides.

Step three – divide to get the final answer.

$$2a+5=15-2a$$
$$+2a+5=+15-2a$$
$$+2a+2a=+15-5$$
$$4a=10$$
$$a=\frac{10}{4}$$
$$a=10\div 4=\underline{2.5}$$

Final step: check, by substituting $a = 2·5$ into the original equation

The left-hand side is $2a + 5$. If we substitute $a = 2·5$, we get $2 \times 2·5 + 5$, which equals 10.

The left-hand side is $15 - 2a$. If we substitute $a = 2·5$, we get $15 - 2 \times 2·5$, which equals 10. These are the same, so our answer must be correct.

If the equation contains brackets, we must multiply them out before continuing.

<u>Example 2 – equation with brackets</u>

Solve algebraically the equation $5(y-1)=3(y+3)$

Solution

Optional first step *– write in "invisible plus signs" in front of anything that does not already have a sign*

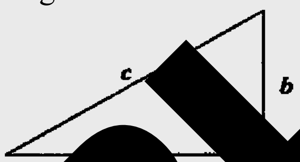
Trigonometry

Pythagoras' Theorem

When you know the length of any two sides of a right angle triangle you can use Pythagoras' Theorem (usually just known as **Pythagoras**) to find the length of the third side without measuring.

Formula: this formula <u>is</u> given on the formula sheet in assessments

Theorem of Pythagoras



$a^2 + b^2 = c^2$

Definition: the hypotenuse is the longest side in a right-angled triangle. In the diagram above, the hypotenuse is c. The hypotenuse is *always* opposite the right angle.

There are three steps to any Pythagoras question:

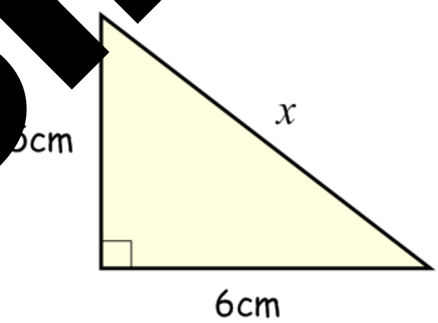
Step One – square the length of the two sides.
Step Two – either add or take away (see below).
Step Three – square root.

Choosing whether to add or take away

- If you are finding the length of the longest side (the hypotenuse), you **add** the squared numbers.
- If you are finding the length of a shorter side, you **take away** the squared numbers.

<u>Example 1 – finding the length of the hypotenuse</u>

Calculate the length of *x* in this triangle.
Do not use a scale drawing.



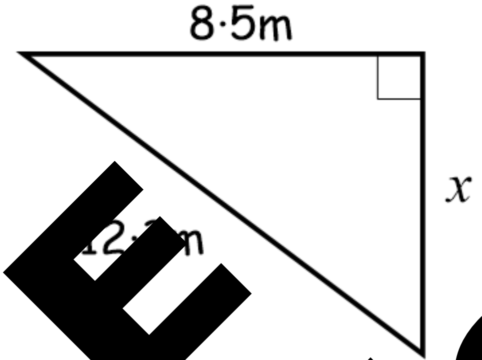
Solution:

We are finding the length of x.
x is the hypotenuse, so we **add**.

$$x^2 = 5^2 + 6^2$$
$$x^2 = 61$$
$$x = \sqrt{61}$$
$$x = 7\cdot81024....$$
$$x = \underline{7\cdot81\text{cm}} \text{ (2 d.p.)}$$

Example 2 – finding the length of a shorter side

Calculate *x*, correct to 1 decimal place. Do not use a scale drawing.



Solution

We are finding the length of x. x is a smaller side, so we **take away**.

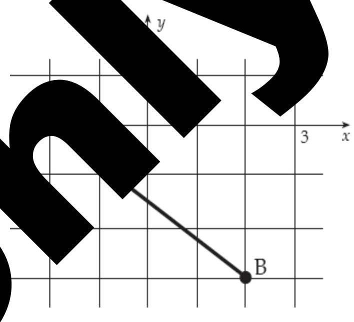
$$x^2 = 12\cdot3^2 - 8\cdot5^2$$
$$x^2 = 79\cdot04$$
$$x = \sqrt{79\cdot04}$$
$$x = 8\cdot8904...$$
$$x = \underline{8\cdot9\text{cm}}$$

In National 4 assessments, the Pythagoras question may [illegible]. On this occasion, there would be a reasoning mark (#2.1) available to you for choosing to use Pythagoras [illegible]. One way to spot them is to look out for the words "**do not use a scale drawing**".

The [illegible] to spot a Pythagoras question is to look for right-angled triangles. However they are not always obvious. The question below is a Pythagoras question although it may not appear to have any right-angled triangle at first.

Example 3 – from coordinates

Calculate the length of the line AB. Do not use a scale drawing.

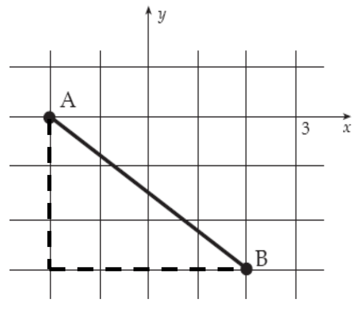


Solution

Can you see the right-angled triangle? Draw lines to complete the triangle.

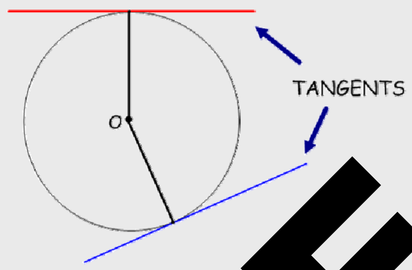
The triangle has sides 3 squares and 4 squares.

Side AB is the hypotenuse, so we **add**



$$AB^2 = 3^2 + 4^2$$
$$AB^2 = 25$$
$$AB = \sqrt{25}$$
$$AB = \underline{5}$$

Definition: a **tangent** to a circle is a line that just touches the edge of the circle.



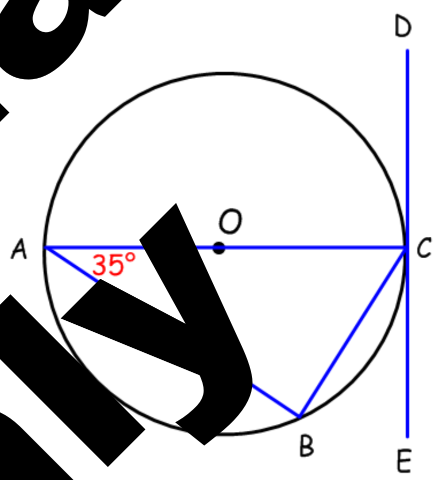
Fact
A tangent always makes a right-angl[illegible] a radius.

A harder example is given bel[illegible]wev[illegible] question is diffe[illegible]he [illegible]ay to get used to them is to pr[illegible]e th[illegible] from p[illegible] Standard Grade Ge[illegible]xam papers, past Intermediate 2[illegible]d textbooks.

Example 2 – harder

The diagram [illegible] circle centre O. DE is a tang[illegible] point C. Angle OAB [illegible]5°.

Calculat[illegible] size of angle BCE.



Sol[illegible]

[illegible]t[illegible]nd a radius meet at r[illegible] ang[illegible] ACD [illegible]ACE are 90°.

T[illegible]e angle in a semi-cir[illegible] is a [illegible]ngle, so ABC is also 90°.

We know two of the [illegible] triangle ABC. Sinc[illegible] we know th[illegible] in a triangle must add to m[illegible] 180°, ang[illegible] AC[illegible] m[illegible]st be 55°.

Finally sin[illegible]ready knew ACE is [illegible], th[illegible]ls us that ACB and BCE [illegible]ust add to make 90°. Therefore angl[illegible]CE is [illegible]°.

[illegible]**al answer:** clearly state that angle BCE is 35° (just marking it on the [illegible]am isn't enough as it doesn't make it clear that you know which angle is angle BCE).

[illegible]argement and Reduction by a Scale Factor

An enlargement of a diagram is another diagram which is the exact same shape (same shape, same angles) except all the lengths have been scaled

The amount of the enlargement is called the **scale factor**. For example:

- an enlargement with a scale factor 2 is double the size of the original. You have to draw the same shape, making every line exactly twice as long.
- an enlargement with a scale factor ½ is half the size of the original. You have to draw the same shape, making every line exactly half as long.

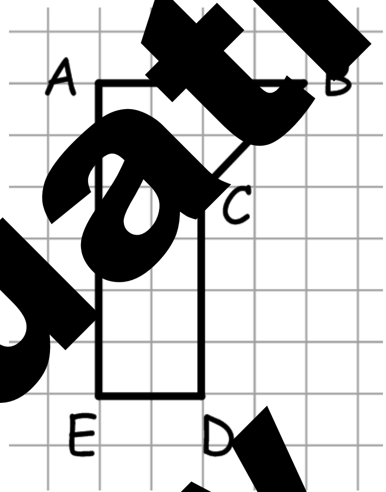
In National 4 assessments, you are expected to draw an enlarged copy of a shape when told a scale factor. The scale factor you will be given will be a fraction, such as $\frac{3}{2}$ or $\frac{5}{4}$.

Deal with a fractional scale factor in the same way [illegible] deal with any fractio[illegible] divide by the number on the bottom and times by the [illegible]r on the top.

Example

Shape ABCDE is sho[illegible] the d[illegible]n the right.

Draw an enla[illegible]men[illegible]e given shape using a scal[illegible]f $\frac{3}{2}$



Solution

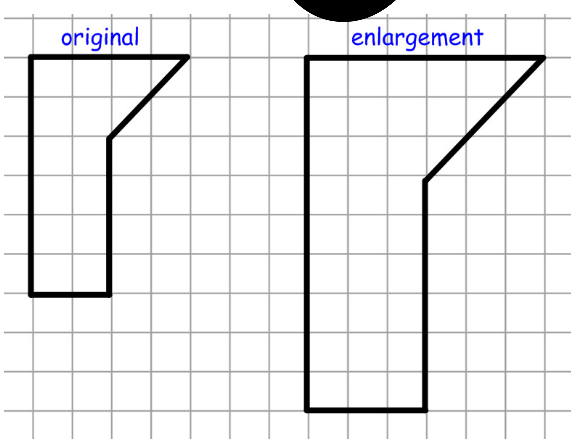
For a scal[illegible]tor of $\frac{3}{2}$, we divide [illegible]gths [illegible] an[illegible]ly by 3 (÷ 2 × 3).

[illegible]xample in the original d[illegible]ine AB [illegible]quares long. If we div[illegible] b[illegible] 2 m[illegible]ltiply by 3, we get 6. So A[illegible] will be 6 squares long [illegible] answ[illegible]

Similarly, CD wi[illegible] 6 s[illegible]res long as well, DE will [illegible] 3 s[illegible] lon[illegible]nd AE will be 9 squares[illegible]

BC will g[illegible]hro[illegible]h the diagonal corners of 3 squ[illegible]

The diagra[illegible]w shows the original [illegible]gra[illegible]d the enlargement side by [illegible]de. The letters have been left out to [illegible]e the[illegible]agram clearer.



Index of Key Words