



www.dynamicmaths.co.uk

# Higher Mathematics

## Q&A Booklet:

### Key Facts to Memorise

#### Ways of using this booklet:

- 1) Write the questions on cards with the answers on the back and **test yourself**.
- 2) **Work with a friend** who is also doing Higher Maths to take turns reading a random question and answering.
- 3) **Ask a friend or family member\*\*** to test you by reading questions (on the left-hand side) to you.

The questions are on the left-hand side of each page and the answers are on the right.

\*\*If the person who is testing you has not done Higher level maths topics recently (or ever!), they may need some help reading the maths symbols, so some mathematical symbols have been written out phonetically (in a smaller bold underlined font) to help them.

Questions are colour coded. The colours are explained on page 2.

**Key** (this is mainly colour coded, so will not work if you are using a black and white printed copy)

Revision from National 5	This knowledge should already be known from National 5 Mathematics and is still required for Higher.
Essential	Learn these facts as soon as possible. They will be essential for classwork.
Important	Learn these facts. You will need them to pass the final exam.
A/B	You will need these facts to pass the course with an A or B but <b>if you are just aiming to pass you may not need them.</b>

All Units: Equations and Graphs	
1) How do you find the y-intercepts of a graph?	Substitute $x = 0$ into its equation.
2) How do you find the roots of a graph?	Substitute $y = 0$ into its equation and solve.
3) What three points do you need to indicate on the sketch of any graph? (if they exist)	<ul style="list-style-type: none"> <li>• Stationary points</li> <li>• Roots</li> <li>• y-intercept</li> </ul>

All Units: Trigonometric Graphs and Equations	
4) For which values of $x$ between 0 degrees and 360 degrees is $\sin x$ ( <u>sine x</u> ) equal to zero?	$0^\circ$ , $180^\circ$ and $360^\circ$
5) For which values of $x$ between 0 degrees and 360 degrees is $\cos x$ equal to zero?	$90^\circ$ and $270^\circ$
6) For which values of $x$ between 0 degrees and 360 degrees is $\tan x$ equal to zero?	$0^\circ$ , $180^\circ$ and $360^\circ$
7) What is the formula for sine in a right-angled triangle?	Opposite over Hypotenuse
8) What is the formula for cos in a right-angled triangle?	Adjacent over Hypotenuse
9) What is the formula for tan in a right-angled triangle?	Opposite over Adjacent
10) What is $\sin 30^\circ$ ? <span style="float: right;"><u>(sine thirty degrees)</u></span>	$\frac{1}{2}$
11) What is $\cos 30^\circ$ ? <span style="float: right;"><u>(cos thirty degrees)</u></span>	$\frac{\sqrt{3}}{2}$ <span style="float: right;"><u>(root three over two)</u></span>
12) What is $\tan 30^\circ$ ? <span style="float: right;"><u>(tan thirty degrees)</u></span>	$\frac{1}{\sqrt{3}}$ <span style="float: right;"><u>(one over root three)</u></span>
13) What is $\sin 45^\circ$ ? <span style="float: right;"><u>(sine forty five degrees)</u></span>	$\frac{1}{\sqrt{2}}$ <span style="float: right;"><u>(one over root two)</u></span>
14) What is $\cos 45^\circ$ ? <span style="float: right;"><u>(cos forty five degrees)</u></span>	$\frac{1}{\sqrt{2}}$ <span style="float: right;"><u>(one over root two)</u></span>
15) What is $\tan 45^\circ$ ? <span style="float: right;"><u>(tan forty five degrees)</u></span>	1
16) What is $\cos 60^\circ$ ? <span style="float: right;"><u>(cos sixty degrees)</u></span>	$\frac{1}{2}$
17) What is $\sin 60^\circ$ ? <span style="float: right;"><u>(sine sixty degrees)</u></span>	$\frac{\sqrt{3}}{2}$ <span style="float: right;"><u>(root three over two)</u></span>
18) What is $\tan 60^\circ$ ? <span style="float: right;"><u>(tan sixty degrees)</u></span>	$\sqrt{3}$ <span style="float: right;"><u>(root three)</u></span>
19) What is $\cos 0^\circ$ ? <span style="float: right;"><u>(cos zero degrees)</u></span>	1
20) What is $\sin 90^\circ$ ? <span style="float: right;"><u>(sine ninety degrees)</u></span>	1
21) What is $\cos 180^\circ$ ? <span style="float: right;"><u>(cos one eighty degrees)</u></span>	-1
22) How do you change from radians to degrees?	Divide by Pi and multiply by 180
23) How do you change from degrees to radians?	Divide by 180 and multiply by Pi
24) What is 180 degrees in radians?	$\pi$ <span style="float: right;"><u>(Pi)</u></span>

25) What is 360 degrees in radians?		<b>(Two Pi)</b>
26) What is 90 degrees in radians?	$\frac{\pi}{2}$	<b>(Pi over two)</b>
27) What is $\pi$ <b>(Pi)</b> radians in degrees?	180°	
28) What is $2\pi$ <b>(Two Pi)</b> radians in degrees?	360°	
29) What is $\frac{\pi}{2}$ <b>(Pi over Two)</b> radians in degrees?	90°	
30) What is the formula for tan?	$\frac{\sin x}{\cos x}$	<b>(sine over cos)</b>
31) How is the range of sin x and cos x restricted?	It is between -1 and 1	
32) How do you find the <b>AMPLITUDE</b> of a trigonometric function from its equation?	It's the number <b>in front of</b> sin, cos or tan	
33) How do you find the <b>FREQUENCY</b> of a trigonometric function from its equation?	It's the number <b>in front of</b> x	

### Applicatons 1.3: Sequences

34) If a limit of a recurrence relation exists, what do we know about the value of 'a'?	It must be between -1 and 1	
35) What is the formula for the <b>LIMIT</b> of a recurrence relation?	$L = \frac{b}{1-a}$	<b>(L equals b over 1 minus a)</b>
36) What sentence do you <b>HAVE TO</b> write for a communication mark when you are calculating a limit?	A limit exists because a equals _____, which is between -1 and 1	
37) How can you find the values that you multiply and add in a recurrence relation when you know the first three terms?	Make two simultaneous equations and solve	

Applications 1.1: The Straight Line	
38) What is the formula for the <b>GRADIENT</b> between two points?	$m = \frac{y_2 - y_1}{x_2 - x_1}$ <b>(y two minus y one over x two minus x one)</b>
39) What is the general formula for the equation of a straight line?	$y - b = m(x - a)$ <b>y minus b equals m brackets x minus a</b>
40) How do you find the gradient of a straight line if you only know its equation?	a) Rearrange to make y the subject b) The gradient is the coefficient of x
41) What is the formula linking the gradient to the angle that a line makes with the x axis?	$m = \tan \theta$ <b>m equals tan theta</b>
42) What is the rule for the gradients of <b>PARALLEL</b> lines?	They are the same
43) What do <b>PERPENDICULAR</b> gradients multiply to give?	-1
44) What two things do you do to find a <b>PERPENDICULAR</b> gradient?	a) Flip it upside down b) Change the sign
45) What is the <b>DISTANCE</b> formula?	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ <b>Square root of x two minus x one squared add y two minus y one squared</b>
46) What are the two properties of the <b>PERPENDICULAR BISECTOR</b> of a line?	a) It goes through the midpoint of the line b) It is perpendicular to the line
47) What are the two properties of the <b>MEDIAN</b> of a triangle through point A?	a) It goes through point A b) It goes through the midpoint of the opposite side
48) What are the two properties of the <b>ALTITUDE</b> of a triangle through point A?	a) It goes through point A b) It is perpendicular to the opposite side
49) How do you find the gradient of an <b>ALTITUDE</b> of a triangle through point A?	a) Find the gradient of the side opposite A b) Flip and change sign
50) How do you find the gradient of the <b>MEDIAN</b> of a triangle through point A?	a) Find the midpoint of the side opposite A b) Find the gradient from the midpoint to A

Applications 1.2: Circles	
51) What does it mean if two circles are <b>CONGRUENT</b> ?	They have the same radius
52) What does it mean if two circles are <b>CONCENTRIC</b> ?	They have the same centre
53) What are the three steps to find the coordinates where a line meets a circle?	a) make $y$ (or $x$ ) the subject of the straight line b) substitute this into the circle equation c) solve the resulting quadratic equation
54) How do you show that a line is a tangent to a circle?	Show there is only one point of intersection <i><b>Alternative answer:</b> show the discriminant of the equation of intersection is zero</i>
55) How do you show that a line and a circle do not intersect?	Show there are no points of intersection <i><b>Alternative answer:</b> show the discriminant of the equation of intersection is negative</i>
56) How do you find the gradient of a tangent to a circle?	It is perpendicular to the radius
57) What is a 'common tangent'?	A line which is a tangent to two circles
58) How can you tell if an equation of a circle is actually an equation of a real circle?	The radius must be positive
59) How do I show that two circles do not intersect?	Show that the distance between the two centres is <b>greater than</b> the sum of the two radii
60) How do I show that two circles touch in one point?	Show that the distance between the two centres is <b>equal to</b> the sum of the two radii

Expressions and Formulae 1.3: Functions and Related Graphs	
61) What is the <b>DOMAIN</b> of a function?	What goes INTO the function
62) What is the <b>RANGE</b> of a function?	What comes OUT of the function
63) What is the turning point of the quadratic function $a(x+p)^2 + q$ ? <u><math>a</math> x plus <math>p</math> squared plus <math>q</math></u>	$(-p, q)$
64) What two possible reasons are there for a value of $x$ not being in the domain of a function?	a) Square rooting a negative number b) Dividing by zero
65) If a function contains a square root, its domain is restricted. The expression under the square root is...	$\geq 0$ <u>(greater than or equal to zero)</u>
66) If a function includes dividing, its domain is restricted. The expressions in the denominator are...	$\neq 0$ <u>(not equal to zero)</u>
67) How has the graph of $y = f(x) + k$ ( <u><math>y</math> equals <math>f</math> of <math>x</math>, plus <math>k</math></u> ) been transformed?	It has been <b>moved up</b> by $k$ units (or down if $k$ is negative)
68) How has the graph of $y = f(x + k)$ ( <u><math>y</math> equals <math>f</math> of <math>x</math> plus <math>k</math></u> ) been transformed?	It has been <b>moved to the left</b> by $k$ units (or to the right if $k$ is negative)
69) How has the graph of $y = kf(x)$ ( <u><math>y</math> equals <math>k</math> of <math>f</math> of <math>x</math></u> ) been transformed?	It has been <b>stretched vertically</b> by scale factor $k$
70) How has the graph of $y = f(kx)$ ( <u><math>y</math> equals <math>f</math> of <math>k</math> of <math>x</math></u> ) been transformed?	It has been <b>compressed horizontally</b> by scale factor $k$
71) How has the graph of $y = -f(x)$ ( <u><math>y</math> equals minus <math>f</math> of <math>x</math></u> ) been transformed?	It has been <b>reflected</b> in the $x$ axis <i>Alternative answer: it has been flipped upside down</i>
72) How has the graph of $y = f(-x)$ ( <u><math>y</math> equals <math>f</math> of minus <math>x</math></u> ) been transformed?	It has been <b>reflected</b> in the $y$ axis <i>Alternative answer: it has been flipped left to right</i>
73) How can you show that two functions $f$ and $g$ are inverses?	Show that $f(g(x)) = x$ ( <u><math>f</math> of <math>g</math> of <math>x</math> equals <math>x</math></u> )
74) What are the two steps for finding the <b>inverse</b> of a function?	a) Switch $x$ and $y$ in the formula b) Change the subject of the formula to $y$

Relationships and Calculus 1.1a: Quadratic Functions	
75) Write down the quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
76) How can we find the minimum/maximum of a quadratic function from its equation?	Complete the square
77) What is formula for the <b>DISCRIMINANT</b> ?	$b^2 - 4ac$ <b>(b squared minus 4 a c)</b>
78) If an equation has <b>real and repeated roots</b> , what do we know about the discriminant?	It is equal to zero (= 0)
79) If an equation has <b>real and distinct roots</b> , what do we know about the discriminant?	It is greater than zero (> 0)
80) If an equation has <b>real roots</b> , what do we know about the discriminant?	It is greater than or equal to zero ( $\geq 0$ )
81) If an equation has <b>no real roots</b> , what do we know about the discriminant?	It is less than zero (< 0)
82) How do we show that a line is tangent to a parabola?	Show they only have one point of intersection  <b>Alternative answer:</b> show the discriminant of the equation of intersection is zero

Relationships and Calculus 1.1b: Quadratic Functions	
83) How do you find the remainder when a polynomial is divided by $x - a$ ? ( <b>x minus a</b> )	Use synthetic division (with $a$ )
84) When doing synthetic division, what do we call the number in the box at the end?	The remainder
85) If a function has a repeated root, what does the graph look like at that point?	It is tangent to the x-axis

**(ignore these next two questions if you were taught to use nature tables instead of the second derivative)**

86) If $\frac{d^2y}{dx^2}$ ( <b>dee two y by dee x squared</b> ) is positive, what does this tell us about a stationary point?	It is a minimum
87) If $\frac{d^2y}{dx^2}$ ( <b>dee two y by dee x squared</b> ) is negative, what does this tell us about a stationary point?	It is a maximum



## Relationships and Calculus 1.3 and Applications 1.4: Differentiation

88) What are the two basic steps for differentiating powers of $x$ ?	a) Multiply by the old power b) Take one away from the power
89) How do you find the <b>DERIVED FUNCTION</b> ?	Differentiate
90) How do you find the <b>DERIVATIVE</b> ?	Differentiate
91) How do you find a <b>RATE OF CHANGE</b> ?	Differentiate
92) How do we find the <b>gradient of a tangent</b> at a particular value of $x$ ?	Differentiate and substitute in $x$
93) What do we know about $\frac{dy}{dx}$ ( <u>dee y by dee x</u> ) if a function is <b>increasing</b> ?	It is greater than zero ( $> 0$ )
94) What do we know about $\frac{dy}{dx}$ ( <u>dee y by dee x</u> ) if a function is <b>decreasing</b> ?	It is less than zero ( $< 0$ )
95) What do we know about $\frac{dy}{dx}$ ( <u>dee y by dee x</u> ) if a function is <b>stationary</b> ?	It is equal to zero ( $= 0$ )
96) What sentence do you <b>HAVE</b> to write for communication marks in an exam in a <b>stationary points</b> question?	A stationary point exists when $\frac{dy}{dx} = 0$ ( <u>dee y by dee x equals zero</u> ) (or $f'(x) = 0$ ( <u>f dash x equals zero</u> ))
97) What are the four types of stationary point?	1. Maximum      3. Rising point of inflection 2. Minimum      4. Falling point of inflection
98) When is a function not differentiable?	If it is not defined at a point
99) What do you get if you differentiate sine?	cos
100) What do you get if you differentiate cos?	-sine
101) What do you have to remember about $x$ when differentiating sine $x$ or cos $x$ ?	$x$ must be in radians and <u>not</u> degrees
102) What are the three steps for differentiating a bracket $(ax + b)^n$ ? <u>(a x plus b to the power n)</u>	1) Multiply by the old power 2) Take one off the power 3) Differentiate the bracket and multiply at the front by your answer

## Relationships and Calculus 1.4 and Applications 1.4: Integration

103) What are the three steps for basic integration?	<ol style="list-style-type: none"> <li>1) Add one to the power,</li> <li>2) Divide by the new power</li> <li>3) Add <math>C</math> (or “add a constant”)</li> </ol>
104) If we have a formula for $\frac{dy}{dx}$ ( <b>dee y by dee x</b> ), how do we find the original formula for $y$ ?	<ol style="list-style-type: none"> <li>a) Integrate</li> <li>b) Substitute in a point on the curve to find <math>C</math></li> </ol>
105) How do we find the area <b>between two</b> curves?	Integral of top take away bottom
106) What must we remember to write at the end of any indefinite integral?	+ $C$
107) What do we have to remember when finding an area below the $x$ -axis?	The answer will be negative, and we have to deal with this appropriately
108) What do you get if you integrate sine $x$ ?	$-\cos x$ plus $C$
109) What do you get if you integrate cos $x$ ?	sine $x$ plus $C$
110) How do you integrate a bracket $(ax + b)^n$ ? <b>(<u>a x plus b to the power n</u>)</b>	<ol style="list-style-type: none"> <li>1. Add one to the power</li> <li>2. Divide by the new power</li> <li>3. Differentiate the bracket and divide by your answer</li> <li>4. Add <math>C</math> (or “add a constant”)</li> </ol>

## Relationships and Calculus 1.2 and Expressions and Functions 1.2: Trigonometric Expressions and Equations

111) In a wave function question, what is the formula for $k$ ?	$k = \sqrt{a^2 + b^2}$ ( <b><u>k equals square root of a squared plus b squared</u></b> )
112) In a wave function question, what is the formula for $a$ ?	$\tan a = \frac{k \sin a}{k \cos a}$ ( <b><u>tan a equals k sine a over k cos a</u></b> )
113) What are the three steps to solve an equation containing $\cos^2 x$ or $\sin^2 x$ ? ( <b><u>cos squared x or sine squared x</u></b> )	<ol style="list-style-type: none"> <li>1) Rearrange to get zero on the right-hand side</li> <li>2) Factorise it</li> <li>3) Use each bracket to make a new equation</li> </ol>
114) If you know solutions to a trig equation between 0 and 360 degrees, how can you find other solutions?	Add or take away multiples of 360 degrees

## Expressions and Functions 1.4: Vectors

### FORMULA SHEET

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta \quad \text{or} \quad \mathbf{a} \bullet \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

115) What are the three steps for finding the magnitude of a vector?	1) Square all the components 2) Add them 3) Square root
116) What is the formula for the components of a vector from coordinate A to coordinate B?	$\mathbf{b} - \mathbf{a}$
117) How do you show that vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel?	Show that there is number $k$ for which $\mathbf{u} = k\mathbf{v}$
118) What does it mean if three points are <b>COLLINEAR</b> ?	They all lie in a straight line
119) How do you show three points are <b>COLLINEAR</b> ?	a) Show two vectors joining them are parallel b) State that they have a common point
120) What is the condition for two vectors to be perpendicular?	Their dot product is zero <b>Alternative:</b> their scalar product is zero
121) How do you show that two vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular?	Show that $\mathbf{a} \bullet \mathbf{b} = 0$ ( <u>a dot b equals zero</u> )
122) What formula do you need to find the angle between two vectors?	$\mathbf{a} \bullet \mathbf{b} =  \mathbf{a}   \mathbf{b}  \cos \theta$ <u>(a dot b equals magnitude of a times magnitude of b times cos theta)</u>
123) What is $\mathbf{a} \bullet \mathbf{a}$ equal to? ( <u>a dot a</u> )	$ \mathbf{a} ^2$ ( <u>the magnitude of a squared</u> )
124) How do you expand the brackets in $\mathbf{a} \bullet (\mathbf{b} + \mathbf{c})$ ? ( <u>a dot brackets b plus c</u> )	$\mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$ ( <u>a dot b plus a dot c</u> )

## Expressions and Formulae 1.1: Exponential and Logarithmic Functions

125) What is $e$ correct to two decimal places?	2.72
126) What is always the value of $\log 1$ ?	0
127) Which two coordinates will an exponential graph (base $a$ ) always pass through?	(0,1) and (1, $a$ )

128) Which two coordinates will a logarithmic graph (base $a$ ) always pass through?	$(1,0)$ and $(a,1)$
129) <b>Laws of logs:</b> When you add two logs with the same base, what happens to the numbers?	Multiply them
130) <b>Laws of logs:</b> When you take away two logs with the same base, what happens to the numbers?	Divide them
131) <b>Laws of logs:</b> How do you deal with a power inside a log?	Bring the power down to the front
132) In an exponential equation $Ae^{kt}$ ( <b>A e to the power k t</b> ), what does $A$ represent?	The starting value
133) In an exponential equation $Ae^{kt}$ ( <b>A e to the power k t</b> ), what does $k$ represent?	The percentage increase/decrease (as a decimal)
134) What two letter abbreviation do we use for the logarithm to the base $e$ ?	LN
135) What is the inverse function of $y = a^x$ ? <b>y equals a to the power of x</b>	$y = \log_a x$ <b>y equals log to the base a of x</b>
136) What is the inverse function of $y = \log_a x$ ? <b>y equals log to the base a of x</b>	$y = a^x$ <b>y equals a to the power of x</b>
137) What is the inverse function of $y = e^x$ ? <b>y equals e to the power of x</b>	$y = \ln x$ <b>y equals L N x</b>
138) What is the inverse function of $y = \ln x$ ? <b>y equals L N x</b>	$y = e^x$ <b>y equals e to the power of x</b>
139) How do you solve an equation where $x$ is in the power? (eg $4^x = 10$ )	Take logs of both sides then bring $x$ down to the front
140) How do you solve an equation with a log on the left-hand side?	Take powers of the right hand side
141) How do you solve an equation containing the number $e$ ?	By taking LN (natural log) of both sides
142) A straight line graph has $\log x$ or $\log y$ on the axes. What are the three steps to find its equation?	a) Find the equation of the line using $y = mx + c$ b) Replace $x$ and $y$ with whatever is on the axes c) Use laws of logs to rearrange